

## Mathematics

## Overview



Effective mathematics education provides students with a balanced instructional program. In such a program, students become proficient in basic computational skills and procedures, develop conceptual understandings, and become adept at problem solving. Standards-based mathematics instruction starts with basic material and increases in scope and content as the years progress. It is like an inverted pyramid, with the entire weight of the developing subject, including readiness for algebra, resting on the foundations built in the early grades.

California recently adopted new standards in mathematics, the Common Core State Standards (CCSS) with California additions. The CCSS are comprised of standards developed by the state-led Common Core State Standards Initiative and material taken from the 1997 California mathematics standards. These new standards will be implemented gradually over the next several years as curriculum frameworks, instructional materials, and assessments based on the CCSS are adopted.

There are many similarities between the CCSS and the 1997 California mathematics standards, but there are also a few noteworthy differences. For instance, the CCSS are organized by "domains" which add grade-level focus and vary slightly by grade. The domains for grade three are Operations and Algebraic Thinking (OA), Number and Operations in Base Ten (NBT), Number and Operations-Fractions (NF), Measurement and Data (MD), and Geometry (G). Also, the CCSS do not include "key standards" as in the 1997 California mathematics standards. Instead the CCSS are designed to have a greater focus at each grade and to develop mathematics topics in depth. In the early grades, the CCSS continue to emphasize concepts necessary for the study of more advanced mathematics in later years. To ensure that students have adequate time to achieve mastery, some of the 1997 California mathematics standards familiar to California's third grade teachers will be taught in different grades after the CCSS are fully implemented.

This section provides an overview of the new CCSS for third grade mathematics, including some highlights of how the third grade curriculum, based on the 1997 California mathematics standards, change with the implementation of the new CCSS. It includes a review of some mathematical concepts for entering third graders to know and guidance on areas of mathematics that may be challenging for some English learners. A complete listing of the CCSS for mathematics can be found at the end of this section. A complete listing of the grade three 1997 California mathematics standards is located on the CDE Content Standards Web page at http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf.

## What Third Grade Students Should Know

Students entering third grade who have met the second grade CCSS for mathematics have an understanding of place value and can read, write, order and compare whole numbers within 1,000 . Students know how to add and subtract (within 1,000 ) and are fluent with these operations within 100. They can use addition and subtraction to solve one- and two-step word problems with unknowns in all positions (within 100) and know from memory all sums of two one-digit numbers.

At the start of third grade, students understand simple concepts of multiplication and division. They can use repeated addition and counting by multiples to demonstrate multiplication and can use repeated subtraction and
equal group sharing to demonstrate division.
Students entering third grade are aware of standard units of measurements and can measure the length of an object using appropriate tools. They can also relate addition and subtraction to length by representing positive whole numbers (from 0) and whole-number sums and differences within 100 on a number line diagram. They know how to model and solve problems involving amounts of money and can use picture graphs and bar graphs to represent and interpret data.

By third grade, students have an understanding of plane and solid geometric shapes and can recognize and describe shapes by various attributes (e.g., the number and shape of faces). They understand the early concepts of area by partitioning rectangles into rows and columns and then counting the number of squares. They can also partition circles and rectangles into two, three and four equal shares and know the associated vocabulary of fractions (e.g., thirds, a third of).

## What Students Learn in Third Grade

Third grade students deepen their understanding of place value and their knowledge of and skill with addition, subtraction, multiplication, and division of whole numbers. Students develop an understanding of fractions as numbers, concepts of area and perimeter of plane figures and attributes of various shapes.

## Operations and Algebraic Thinking

The 1997 California mathematics standards and the CCSS foster an understanding of the relationship between multiplication and division. Third graders fluently multiply and divide (within 100) and use simple multiplication and division to solve word problems (using drawings and equations with a symbol for the unknown number to represent the problem). They understand division as an unknown-factor problem (e.g., find 32 $\div 8$ by finding the number that makes 32 when multiplied by 8 ) and use the inverse relationship between multiplication and division to compute and check results. Students apply their knowledge and skills with the four operations (addition, subtraction, multiplication and division) to solve word problems.

By the end of third grade students will know from memory all products of

Students apply their knowledge and skills with the four operations (addition, subtraction, multiplication and division) to solve word problems. numbers from 1 to 9 (the multiplication tables for 2 s and 5 s are introduced at grade two in the 1997 California standards). Students discover that the associative and commutative laws reduce the number of multiplication facts they need to learn. For example, if a student knows $5 \times 9$, then they also know $9 \times 5$.

With full implementation of the CCSS, multiplication and division of a whole number (with up to four digits) and a one-digit whole number, (e.g., $3,671 \times 3=\ldots$ or $1,035 \div 5=\ldots$ ) will be covered at grade four, a grade three topic in the 1997 California standards.

## Number and Operations in Base Ten

In both the 1997 California mathematics standards and the CCSS, third grade students extend their place value understanding to include numbers with four digits. They round whole numbers to the nearest 10 or 100, a critical prerequisite for working estimation problems. With full implementation of the CCSS, rounding numbers to the nearest thousands will be covered at grade four.

Students also apply their understanding of place value as they fluently add and subtract (within 1000) in which regrouping or composing a ten (i.e., carrying and borrowing) is required in more than one column.

Students may need extra practice solving problems requiring regrouping across columns with zeros, which can be confusing. With full implementation of the CCSS, addition and subtraction with two whole numbers (within $1,000-10,000$ ) will be covered at grade four.

## Number and Operations-Fractions

Student proficiency with fractions is essential to success in algebra at later grades. In grade three, both the 1997 California mathematics standards and the CCSS develop an understanding of fractions as numbers. Students use visual fractional models to represent fractions as parts of a whole. They also use visual models and a number line to represent, explain, and compare unit fractions (fractions with a numerator 1), equivalent fractions (e.g., $1 / 2=2 / 4$ ), whole numbers as fractions (e.g., $3=3 / 1$ ), and fractions with the same numerator or the same denominator.

With full implementation of the CCSS, third grade students will learn to recognize, name, and compare fractions (a grade two topic in the 1997 California mathematics standards) and use a number line to represent positive fractions (a grade four topic in the 1997 California mathematics standards). Also, operations with decimals will be introduced at grade five (a grade three topic in the 1997 California mathematics standards).

## Measurement and Data



In grade three, the 1997 California mathematics standards and the CCSS focus on measurement. Students measure lengths (using a ruler), liquid volume (using standard units), and the area of plane figures (by counting unit squares). Students demonstrate an understanding of fractions as they measure lengths using rulers marked with halves and fourths of an inch. Students solve problems involving the perimeter of polygons. They relate the concept of area to the operations of multiplication and division and show that the area of a rectangle can be found by multiplying the side lengths.

With full implementation of the CCSS, the probability of a chance event and simple predictions, a grade three topic in the 1997 California mathematics standards, will be introduced and developed at grade seven. Also, simple unit conversions, for example centimeters to meters, a grade three topic in the 1997 California mathematics standards, will be studied at grade five as students use conversions to solve problems.

## Geometry

In grade three, the 1997 California mathematics standards and the CCSS focus on the attributes of shapes. Students compare common geometric shapes (e.g., rectangles and quadrilaterals) based on common attributes (e.g., having four sides). Students also relate their work with fractions to geometry as they partition shapes into parts with equal areas and represent each part as a unit fraction of the whole.

With full implementation of the CCSS, identifying right angles in geometric shapes, a grade three topic in the 1997 California mathematics standards, will be covered at grade four, beginning with right triangles.

## Support for English Learners

Students need to develop knowledge of mathematics as a language. However, the academic language of mathematics instruction and the specialized vocabulary of mathematics can create particular challenges for English learners.

The language of mathematics is very precise compared with the English used in common discourse. English learners need opportunities to develop their knowledge of the features of language that are used to teach mathematics, such as semantics (how to translate the words of a problem into a symbolic representation), syntax (the order of words and phrases), and mathematical discourse (writing or talking about mathematical terms, concepts, etc.). The specialized vocabulary of mathematics should be explicitly taught and reinforced throughout the year.

These areas can create special challenges for English learners in the early grades:

- At an early stage students may have difficulty with such English words as first, second, last, before, every, each, more, and equal. Students may be unfamiliar with sum, difference, solve, length, and value.
- The different meanings of multiple-meaning words should be explicitly taught. These words may have a meaning in common discourse that is different from the meaning in mathematics, such as table or face (as in the face of a clock).
- The place values of some of the numbers between 10 and 20 are not obvious from their names (e.g., the number 16 is called sixteen in English, but ten plus six in other languages).
- The narrative descriptions of a word problem can require language skills that students have not yet mastered, particularly when the language of a word problem is ambiguous or includes idioms (e.g., "a dime a dozen"), comparatives (greater than, less than, most often, least often), or position words (behind, below, in front of, to the right or left of).

Instruction in mathematics should be promoted despite low literacy or limited proficiency in the English language, along with critical thinking and analysis skills. Specially designed academic instruction in English (SDAIE) strategies can provide valuable instructional strategies to meet the needs of English learners. For additional resources to support the teaching of English learners, go to the CDE English Learners Web page at http://www.cde.ca.gov/sp/el/.

## Transition to Common Core State Standards

The following chart highlights a few topics that will continue to be addressed at the grade level and some of the changes to be considered as California progresses toward full implementation of the grade three CCSS for mathematics. The chart includes the column heading "Overview of Standards." For the 1997 California mathematics standards, this information is from the "strands" (e.g., Number Sense) and the "overarching" standards (e.g., Number Sense 1.0) at grade three. For the CCSS, the column lists the "domains" (e.g., Operations and Algebraic Thinking) and the "cluster headings" for the standards (e.g., Represent and solve problems involving multiplication and division) at grade three.

The chart does not, and is not intended to, illustrate all of the differences between the two sets of standards-it is merely a beginning point for more in-depth discussion by teachers and other educators on how instruction may change.

The transition chart is followed by a complete set of the CCSS for grade three and then a table of the CCSS domains for kindergarten through grade five.
Mathematics: Grade Three

## Overview of Standards

1997 California Mathematics Standards
Algebra and Functions

- Students select appropriate symbols, operations, and properties to represent, describe, simplify, and solve simple numbe relationships.
- Students represent simple functional
relationships.
Number Sense
numbers.
- Students calculate and solve problems
involving addition, subtraction,
multiplication, and division.
- Students understand the relationship between
whole numbers, simple fractions, and decimals.
${ }^{1}$ Note: The 1997California standards will continue to be assessed through the STAR system (in grades 2-11) until at least 2014.
${ }^{2}$ The $\boldsymbol{\Delta}$ symbol indicates all or part of a concept in the 1997 California mathematics standards has moved to a higher grade in the CCSS; the $\boldsymbol{\nabla}$ symbol indicates movement to a lower grade. No symbol indicates a concept will continue to be taught at the current grade level.

|  |  | grade three to grade four in the CCSS). $\Delta$ |
| :---: | :---: | :---: |
| Measurement and Geometry <br> - Students choose and use appropriate units and measurement tools to quantify the properties of objects. <br> - Students describe and compare the attributes of plane and solid geometric figures and use their understanding to show relationships and solve problems. | Number and Operations - Fractions <br> - Develop understanding of fractions as numbers. | - Understand a fraction $1 / b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\mathrm{a} / \mathrm{b}$ as the quantity formed by a parts of size $1 / \mathrm{b}$ (introduction to fractions moves from grade two to grade three in the CCSS). <br> - Represent fractions on a number line diagram (moves from grade four to grade three in the CCSS). <br> - Compare two fractions (with the same numerator or the same denominator) and recognize and generate simple equivalent fractions (e.g., $1 / 2=2 / 4$ and $4 / 6=$ 2/3) using visual fractional models. <br> - Add and subtract simple fractions (moves from grade three to grade four in the CCSS). <br> - Operations with decimals (moves from grade three to grade four in the CCSS). |
| Statistics, Data Analysis, and Probability <br> - Students conduct simple probability experiments by determining the number of possible outcomes and make simple predictions. | Measurement and Data <br> - Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. <br> - Represent and interpret data. <br> - Geometric measurement: understand concepts of area and relate area to multiplication and to addition. <br> - Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. | - Measure and estimate liquid volumes and masses of objects using standard units (work with unit conversions moves from grade three to grade five in the CCSS). 4 <br> - Represent data in graphs (scaled picture graph or bar graph) and use the information to solve problems (use of data from picture graphs to solve addition and subtraction problems moves from grade two to grade three in the CCSS). <br> - Measure areas by counting unit squares and show that the area is the same as would be found by multiplying the side lengths. <br> - Solve problems involving perimeters of polygons. <br> - Introduce probability of a chance event and simple predictions (moves from grade three to grade |


|  |  | seven in the CCSS). $\triangle$ |
| :---: | :---: | :---: |
|  | Geometry <br> - Reason with shapes and their attributes. | - Understand shapes may share attributes which can define a larger category. <br> - Partition shapes into parts with equal areas to represent a unit fraction of the whole. <br> - Identify right angles in geometric shapes (moves from grade three to grade four in the CCSS). |
| Mathematical Reasoning <br> - Students make decisions about how to approach problems. <br> - Students use strategies, skills, and concepts in finding solutions. <br> - Students move beyond a particular problem by generalizing to other situations. | Standards for Mathematical Practice <br> 1. Make sense of problems and persevere in solving them. <br> 2. Reason abstractly and quantitatively. <br> 3. Construct viable arguments and critique the reasoning of others. <br> 4. Model with mathematics. <br> 5. Use appropriate tools strategically. <br> 6. Attend to precision. <br> 7. Look for and make use of structure. <br> 8. Look for and express regularity in repeated reasoning. | - The CCSS include Standards for Mathematical Content (different at each grade) and Standards for Mathematical Practice (recurring throughout the grades). <br> - To master the grade level content, students will need to rely on their understanding of a concept and not only on procedures. Standards for Mathematical Practice define how students develop mathematical understanding as they make sense of a problem, reason abstractly, construct arguments, model with mathematics, use tools strategically, attend to precision, and look for structure and repeated reasoning. <br> - Standards for Mathematical Content that set an expectation of "understanding" are potential points of intersections between these standards and the Standards for Mathematical Practice. <br> - Standards for Mathematical Practice are similar to the previous 1997 California Mathematical Reasoning standards and should be evident throughout future curricula, assessments and professional development. |

## The Standards

The CCSS that follow are the pre-publication version of the standards prepared by the Sacramento County Office of Education (SCOE), updated on October 21, 2010. Content that is unique to California and was added to the multi-state common core standards is in bold typeface. The SCOE document is available online at http://www.scoe.net/castandards/agenda/2010/math_ccs_recommendations.pdf (Outside Source). These grade three CCSS for Mathematics were adopted by the California State Board of Education on August 2, 2010.

A complete listing of the grade three 1997 California mathematics standards is located on the CDE Web page at http://www.cde.ca.gov/be/st/ss/documents/mathstandard.pdf.

| Common Core State Standards <br> with California Additions <br> Mathematics - Grade Three |
| :--- | :--- |
| Operations and Algebraic Thinking (3.0A) |
| Represent and solve problems involving multiplication and division. |

[^0]| Multiply and divide within 100. |  |
| :---: | :---: |
| 7. | Fluently multiply and divide within 100 , using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5=40$, one knows $40 \div 5=8$ ) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. |
| Solve problems involving the four operations, and identify and explain patterns in arithmetic. |  |
| 8. | Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. ${ }^{3}$ |
| 9. | Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends. |
| Number and Operations in Base Ten (3.NBT) |  |
| Use place value understanding and properties of operations to perform multi-digit arithmetic. ${ }^{4}$ |  |
| 1. | Use place value understanding to round whole numbers to the nearest 10 or 100. |
| 1.1 | Understand that the four digits of a four-digit number represent amounts of thousands, hundreds, tens, and ones; e.g. $3,706=3000+700+6=3$ thousands, 7 hundreds, 0 tens, and 6 ones. |
| 2. | Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. |
| 3. | Multiply one-digit whole numbers by multiples of 10 in the range $10-90$ (e.g., $9 \times 80,5 \times 60$ ) using strategies based on place value and properties of operations. |
| Number and Operations-Fractions (3.NF) ${ }^{5}$ |  |
| Develop understanding of fractions as numbers. |  |
| 1. | Understand a fraction $1 / b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a / b$ as the quantity formed by a parts of size $1 / b$. |
| 2. | Understand a fraction as a number on the number line; represent fractions on a number line diagram. <br> a. Represent a fraction $1 / b$ on a number line diagram by defining the interval from 0 to 1 as |

[^1]|  | the whole and partitioning it into $b$ equal parts. Recognize that each part has size $1 / b$ and that the endpoint of the part based at 0 locates the number $1 / b$ on the number line. <br> b. Represent a fraction $a / b$ on a number line diagram by marking off $a$ lengths $1 / b$ from 0 . Recognize that the resulting interval has size $a / b$ and that its endpoint locates the number $a / b$ on the number line. |
| :---: | :---: |
| 3. | Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. <br> a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. Recognize that equivalencies are only valid when the two fractions refer to the same whole. <br> b. Recognize and generate simple equivalent fractions, e.g., $1 / 2=2 / 4,4 / 6=2 / 3$ ). Explain why the fractions are equivalent, e.g., by using a visual fraction model. <br> c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3=3 / 1$; recognize that $6 / 1=6$; locate $4 / 4$ and 1 at the same point of a number line diagram. <br> d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. <br> e. Know and understand that 25 cents is a $1 / 4$ of a dollar, 50 cents is $1 / 2$ of a dollar, and 75 cents is $3 / 4$ of a dollar. |
|  | Measurement and Data (3.MD) |
| Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. |  |
| 1. | Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram. |
| 2. | Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and English Units (oz, lb.), and liters (l). ${ }^{6}$ Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. ${ }^{7}$ |
| Represent and interpret data. |  |

[^2]| 3. | Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph might represent 5 pets. |
| :---: | :---: |
| 4. | Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units- whole numbers, halves, or quarters. |
| Geometric measurement: understand concepts of area and relate area to multiplication and to addition. |  |
| 5. | Recognize area as an attribute of plane figures and understand concepts of area measurement. <br> a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. <br> b. A plane figure which can be covered without gaps or overlaps by $n$ unit squares is said to have an area of $n$ square units. |
| 6. | Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units). |
| 7. | Relate area to the operations of multiplication and addition. <br> a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. <br> b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. <br> c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths $a$ and $b+c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning. <br> d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into nonoverlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems. |
| Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. |  |
| 8. | Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters. |

## Geometry (3.G)

## Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as $1 / 4$ of the area of the shape.

## Standards for Mathematical Practice

Integrated throughout the CCSS

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

## CCSS Domains

The CCSS are organized by domains. The table lists the domains for grades kindergarten through grade eight. The table identifies which domains are addressed in kindergarten through grade five (an " X " indicates the domain addressed at a grade level). The shaded rows indicate domains to be covered at later grades.

| Domains | Kindergarten | Grade <br> One | Grade <br> Two | Grade <br> Three | Grade <br> Four | Grade <br> Five |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Counting and Cardinality (CC) | $\mathbf{X}$ |  |  |  |  |  |
| Operations and Algebraic Thinking (OA) | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| Number and Operations in Base Ten (NBT) | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| Measurement and Data (MD) | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| Geometry (G) | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| Number and Operations - Fractions (NF) |  |  |  | $\mathbf{X}$ | $\mathbf{X}$ | $\mathbf{X}$ |
| Ratios and Proportional Relationships (RP) |  |  |  |  |  |  |
| The Number System (NS) |  |  |  |  |  |  |
| Expressions and Equations (EE) |  |  |  |  |  |  |
| Statistics and Probability (SP) |  |  |  |  |  |  |
| Functions (F) |  |  |  |  |  |  |


[^0]:    ${ }^{1}$ See Glossary, Table 2.
    ${ }^{2}$ Students need not use formal terms for these properties.

[^1]:    ${ }^{3}$ This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).
    ${ }^{4}$ A range of algorithms may be used.
    ${ }^{5}$ Grade 3 expectations in this domain are limited to fractions with denominators $2,3,4,6$, and 8 .

[^2]:    ${ }^{6}$ Excludes compound units such as $\mathrm{cm}^{3}$ and finding the geometric volume of a container.
    ${ }^{7}$ Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

