Delaware's

Common Core State Standards for Mathematics

Grade 8 Assessment Examples

Delaware
Department
of Education

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Delaware's Common Core State Standards for 8th Grade Mathematics

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Grade 8 Mathematics – Unpacking the Delaware Common Core State Standards

This document is designed to help understand the Common Core State Standards (CCSS) in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

What Is the Purpose of This Document?

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

References

This document contains explanations and examples that were obtained from State Departments of Education for Utah, Arizona, North Carolina, Kansas, and New York with permission.

How Do I Send Feedback?

This document is helpful in understanding the CCSS but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education via rfry@doe.k12.de.us, and we will use your input to refine this document.

Number and Quantity: The Number System (NS)

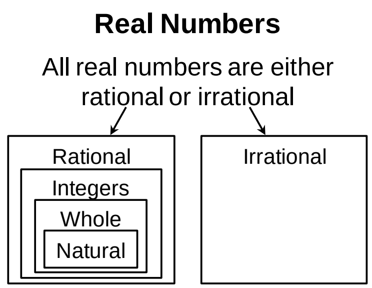
Know that there are numbers that are not rational, and approximate them by rational numbers.

Standard

8.NS.1 – Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

Resources: Students can use graphic organizers to show the relationship between the subsets of the real number system.

Example:



- Any number that can be expressed as a fraction is a rational number. A rational number is of the form $\frac{a}{b}$, where a and b are both integers and b is not 0. Students recognize that the decimal equivalent of a fraction will either terminate or repeat.

  Change $0.\overline{4}$ to a fraction.
  - Let $x = 0.444444.....$
  - Multiply both sides so that the repeating digits will be in front of the decimal. In this example, one digit repeats so both sides are multiplied by 10, giving $10x = 4.444444.....$ Subtract the original equation from the new equation.

    $10x = 4.4444444.....$

    $x = 0.444444.....$

    $x = \frac{4}{9}$

    Students can investigate repeating patterns that occur with denominators 9, 99, or 11.

    $\frac{4}{9}$ is equivalent to $0.\overline{4}$, $\frac{5}{9}$ is equivalent to $0.\overline{5}$, etc.

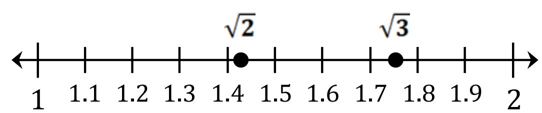
Standard
8.NS.2 – Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of* $\sqrt{2}$*, show that* $\sqrt{2}$ *is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Explanation: Students can approximate square roots and locate rational and irrational numbers on the number line. Students compare and order rational numbers. Additionally, students understand that the value of a square root can be approximated between integers and that non-perfect square roots are irrational.

Examples:

- Approximate the value of $\sqrt{5}$ to the nearest tenth.
  - Solution: Students start with a rough estimate based upon perfect squares. $\sqrt{5}$ falls between 2 and 3 because 5 falls between $2^2 = 4$ and $3^2 = 9$. The value will be closer to 2 than 3.

    To find an approximation of 28, first determine the perfect squares 28 is between, which would be 25 and 36. The square roots of 25 and 36 are 5 and 6, respectively, so we know that 28 is between 5 and 6. Since 28 is closer to 25, an estimate of the square root would be closer to 5.
- Compare $\sqrt{2}$ and $\sqrt{3}$ by estimating their values, plotting them on a number line, and making comparative statements.



  - Solution: Statements for the comparison could include:
    - $\sqrt{2}$ is approximately 0.3 less than $\sqrt{3}$.
    - $\sqrt{2}$ is between the whole numbers 1 and 2.
    - $\sqrt{3}$ is between 1.7 and 1.8.

Expressions and Equations (EE)

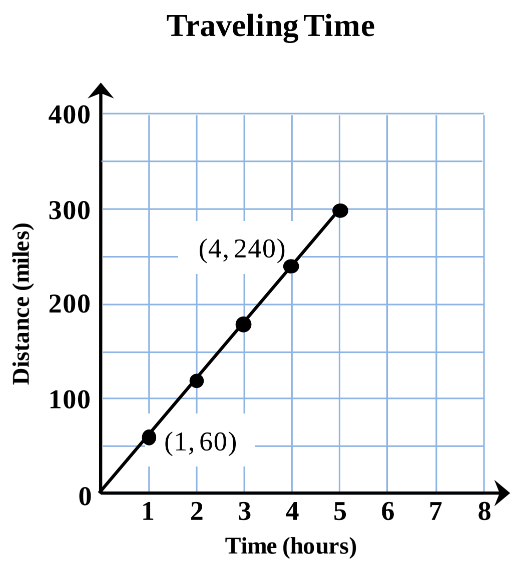
Work with radicals and integer exponents.

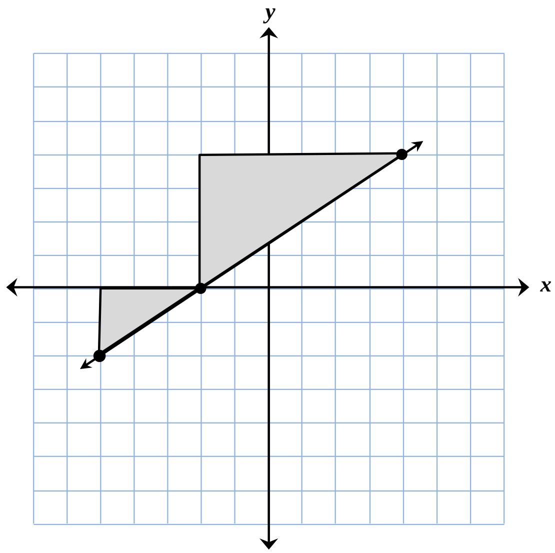
Standard
8.EE.1 – Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example,* $3^2 \times 3^{-5} = 3^{-3} = \frac{1}{3^3} = \frac{1}{27}$. *Explanation:* Integer (positive and negative) exponents are further used to generate expressions when multiplying, dividing or raising a power to a power. When multiplying like bases, add the exponents; when dividing like bases, subtract the exponents; and when raising a power to a power, multiply the exponents. *Examples:* • $\frac{4^3}{5^2} = \frac{64}{25}$ • $\frac{4^3}{4^7} = 4^{3-7} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$ • $\frac{4^{-3}}{5^2} = 4^{-3} \times \frac{1}{5^2} = \frac{1}{4^3} \times \frac{1}{5^2} = \frac{1}{64} \times \frac{1}{25} = \frac{1}{16,000}$
8.EE.2 – Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational. *Explanation:* Students recognize that squaring a number and taking the square root of a number are inverse operations; likewise, cubing a number and taking the cube root are inverse operations. Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students understand that in geometry a square root is the length of the side of a square and a cube root is the length of the side of a cube. The value of x for square root and cube root equations must be positive. *Examples:* • $3^2 = 9$ and $\sqrt{9} = \pm 3$ • $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ • Solve: $x^2 = 9$ Solution: $\sqrt{x^2} = \pm\sqrt{9}$ $x = \pm 3$ • Solve: $x^3 = 8$ $\sqrt[3]{x^3} = \sqrt[3]{8}$ $x = 2$

Standard
8.EE.3 – Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.* *Explanation:* Students express numbers in scientific notation. Students compare and interpret scientific notation quantities in the context of the situation. If the exponent increases by one, the value increases 10 times. Students understand the magnitude of the number being expressed in scientific notation and choose an appropriate corresponding unit. For example, 3×10^8 is equivalent to 30 million, which represents a large quantity. Therefore, this value will affect the unit chosen. *Example:* • An ant has a mass of approximately 4×10^{-3} grams and an elephant has a mass of approximately 8 metric tons. a. How many ants does it take to have the same mass as an elephant? b. An ant is 10^{-1} cm long. If you put all these ants from your answer to part a. in a line front to back, how long would the line be? Find two cities in the United States that are a similar distance apart to illustrate this length. Note: 1 kg = 1000 grams, 1 metric ton = 1000 kg, 1 m = 100 cm, 1 km = 1000 m
8.EE.4 – Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology. *Explanation:* Students can convert decimal forms to scientific notation and apply rules of exponents to simplify expressions. In working with calculators or spreadsheets, it is important that students recognize scientific notation. Students should recognize that the output of $2.45E + 23$ is 2.45×10^{23} and $3.5E - 4$ is 3.5×10^{-4}. Students enter scientific notation using E or EE (scientific notation), × or • (multiplication), and ^ (exponent) symbols. Students use the laws of exponents to multiply or divide numbers written in scientific notation. *Example:* • The following headline appeared in a newspaper: "Every day 7% of Americans eat at Giantburger restaurants" Decide whether this headline is true using the following information: ▪ There are about 8×10^3 Giantburger restaurants in America ▪ Each restaurant serves on average 2.5×10^3 people every day ▪ There are about 3×10^8 Americans Explain your reasons and clearly show your work.

Understand the connections between proportional relationships, lines, and linear equations.

Standard
8.EE.5 – Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.* *Explanation:* Using graphs of experiences that are familiar to students increases accessibility and supports understanding and interpretation of proportional relationship. Students are expected to both sketch and interpret graphs. Students build on their work with unit rates and proportional relationships to compare graphs, tables, and equations of proportional relationships. Students identify the unit rate (or slope) in graphs, tables, and equations to compare two or more proportional relationships. *Example:* • Compare the scenarios to determine which represents a greater speed. Include a description of each scenario including the unit rates in your explanation.

Scenario 1	Scenario 2
	$y = 50x$ x is time in hours y is distance in miles

Standard
8.EE.6 – Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation *y* = *mx* for a line through the origin and the equation *y* = *mx* + *b* for a line intercepting the vertical axis at *b*. *Resources:* Triangles are similar when there is a constant rate of proportion between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line. The triangle between A and B has a vertical height of 2 and a horizontal length of 3. The triangle between B and C has a vertical height of 4 and a horizontal length of 6. The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents the slope of the line. Students write equations in the form $y = mx$ for lines going through the origin, recognizing that m represents the slope of the line. Students write equations in the form $y = mx + b$ for lines not passing through the origin, recognizing that m represents the slope and b represents the y-intercept. 

Analyze and solve linear equations and pairs of simultaneous linear equations.

Standard
8.EE.7 – Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form *x* = *a*, *a* = *a*, or *a* = *b* results (where *a* and *b* are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms. *Explanations:* • Students solve one-variable equations with the variables being on both sides of the equals sign. Students recognize that the solution to the equation is the value(s) of the variable, which make a true equality when substituted back into the equation. Equations shall include rational numbers, distributive property, and combining like terms. • Equations have one solution when the variables do not cancel out. For example, $10x - 23 = 29 - 3x$ can be solved to $x = 4$. This means that when the value of x is 4, both sides will be equal. If each side of the equation is treated as a linear equation and graphed, the solution of the equation represents the coordinates of the point where the two lines would intersect. In this example, the ordered pair would be $(4, 17)$. $10 \bullet 4 - 23 = 29 - 3 \bullet 4$ $40 - 23 = 29 - 12$ $17 = 17$ • Equations having no solution have variables that will cancel out and constants that are not equal. This means that there is not a value that can be substituted for x that will make the sides equal. For example, the equation $-x + 7 - 6x = 19 - 7x$, can be simplified to $-7x + 7 = 19 - 7x$. If $7x$ is added to each side, the resulting equation is $7 = 19$, which is not true. No matter what value is substituted for $x,$ the final result will be $7 = 19$. If each side of the equation were treated as a linear equation and graphed, the lines would be parallel. • An equation with infinitely many solutions occurs when both sides of the equation are the same. Any value of x will produce a valid equation. For example the following equation, when simplified will give the same values on both sides. $-3(6\text{a} - 1) = 9(\frac{1}{3} - 2a)$ $-18a + 3 = 3 - 18a$ If each side of the equation are treated as a linear equation and graphed, the graph would be the same line.

Standard

8.EE.8 – Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.*

c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Explanations:

- Systems of linear equations can also have one solution, infinitely many solutions, or no solutions. Students will discover these cases as they graph systems of linear equations and solve them algebraically.
- A system of linear equations whose graphs meet at one point (intersecting lines) has only one solution, the ordered pair representing the point of intersection. A system of linear equations whose graphs do not meet (parallel lines) has no solutions, and the slopes of these lines are the same. A system of linear equations whose graphs are coincident (the same line) has infinitely many solutions, the set of ordered pairs representing all the points on the line.
- By making connections between algebraic and graphical solutions, students are able to make sense of their solutions. Students need opportunities to work with equations that include whole number and/or decimals/fractions.

Examples:

- Find x and y using elimination and then using substitution.
  - $3x + 4y = 7$
  - $-2x + 8y = 10$
- Plant A and Plant B are on different watering schedules. This affects their rate of growth. Compare the growth of the two plants to determine when their heights will be the same.

  Let W = number of weeks

  Let H = height of the plant after W weeks

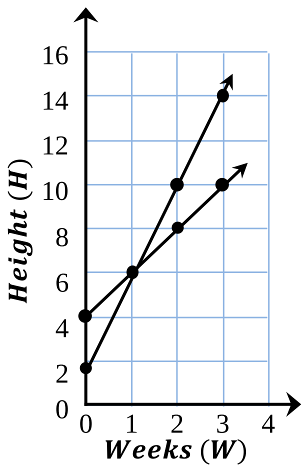
Plant A		
$\boldsymbol{W}$	$\boldsymbol{H}$	$(\boldsymbol{W},\boldsymbol{H})$
0	4	(0,4)
1	6	(1,6)
2	8	(2,8)
3	10	(3,10)

Plant B		
$\boldsymbol{W}$	$\boldsymbol{H}$	$(\boldsymbol{W},\boldsymbol{H})$
0	2	(0,2)
1	6	(1,6)
2	10	(2,10)
3	14	(3,14)

Standard
8.EE.8 Examples (continued)

- Given each set of coordinates, graph their corresponding lines.

  Solution:



- Write an equation that represents the growth rate of Plant A and Plant B.

  Solution:

  Plant A $H = 2W + 4$

  Plant B $H = 4W + 2$

- At which week will the plants have the same height?

Plant A	Plant B
$H = 2W + 4$	$H = 4W + 2$
$H = 2(1) + 4$	$H = 4(1) + 2$
$H = 6$	$H = 6$

After one week, the height of Plant A and Plant B are both 6 inches.

Functions (F)

Define, evaluate, and compare functions.

Standard
8.F.1 – Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required in Grade 8.)

Explanations: Using y to stand for the output and x as input, we can represent this function with the equation $y = x^2 + 5x + 4$, and the graph of the equation is the graph of the function. Students distinguish between functions and non-functions using equations, graphs, and tables. Non-functions occur when there is more than one y-value associated with any x-value. Students are not yet expected to use function notation such as $f(x) = x^2 + 5x + 4$. "Function machine" pictures are useful for helping students imagine input and output values.

Examples:

- Use the data in each input/output table to determine a two-step rule.

a.

Input	Output
2	14
10	30
1	12
0	10
-5	0
1.5	13
$\frac{1}{3}$	$10\frac{2}{3}$
$\frac{1}{2}$	11
5	20
101	212

b.

Input	Output
2	3
0	-3
-2	-9
5	12
10	27
100	297
$\frac{1}{2}$	-1.5
3	6
$\frac{1}{4}$	$-\frac{9}{4}$ or $-2\frac{1}{4}$
-1	-6

c.

Input	Output
5	3.5
2	2
0	1
10	6
-5	-1.5
-2	0
-10	-4
$\frac{1}{2}$	$1\frac{1}{4}$
100	51
-100	-49

d.

Input	Output
10	49
0	-1
1	4
3	14
5	24
100	499
$\frac{1}{2}$	$1\frac{1}{2}$
4	19
-4	-21
12	59

Standard

8.F.1 Examples (continued)

- Following is a set of inputs and outputs for a mystery machine:

a.

Input	Output
3	10
10	17
1	8
0	7
−5	2
1.5	8.5
$\frac{1}{3}$	$7\frac{1}{3}$

b.

Input	Output
3	10
10	24
1	6
0	4
−5	−6
1.5	7
$\frac{1}{3}$	$4\frac{2}{3}$

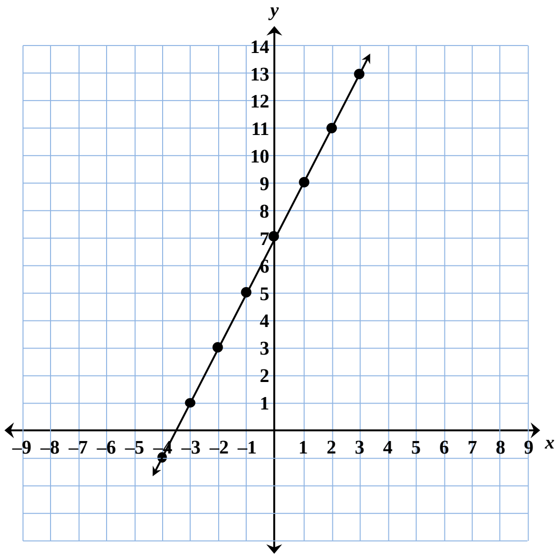
Create **two** equivalent two-step rules.

Resources: If you have access to a computer, show students how to build one- and two-step machines using a spreadsheet. Have students go back through this activity and see if they can build a spreadsheet for each machine.

Standard
8.F.2 – Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

Examples:

- Compare the two linear functions listed below and determine which equation represents a greater rate of change (slope of the line or the coefficient of x).

Function 1:	Function 2:
	The function whose input x and output y are related by: $y = 3x + 7$

Standard

8.F.2 Examples (continued)

- Compare the two linear functions listed below and determine which has a negative slope.
  - Function 1: Gift Card

    Samantha starts with $20 on a gift card for the bookstore. She spends $3.50 per week to buy a magazine. Let y be the amount remaining as a function of the number of weeks, x.

x	y
0	20.00
1	16.50
2	13.00
3	9.50
4	6.00

  - Function 2:

    The school bookstore rents graphing calculators for $5 per month. It also collects a non-refundable fee of $10 for the school year. Write the rule for the total cost (c) of renting a calculator as a function of the number of months (m).
  - Solution:
    - Function 1 is an example of a function whose graph has negative slope. Samantha starts with $20 and spends money each week. The amount of money left on the gift card decreases each week. The graph has a negative slope of -3.5, which is the amount the gift card balance decreases with Samantha's weekly magazine purchase.
    - Function 2 is an example of a function whose graph has positive slope. Students pay a yearly, nonrefundable fee for renting the calculator and pay $5 for each month they rent the calculator. This function has a positive slope of 5, which is the amount of the monthly rental fee. An equation for example 2 could be $c = 5m + 10$.

8.F.3 – Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Explanation: Students use equations, graphs, and tables to categorize functions as linear or non-linear. Students recognize that points on a straight line will have the same rate of change between any two of the points.

Example:

- $y = -2x^2 + 3$ non-linear
- $y = 2x$ linear
- $A = \pi r^2$ non-linear
- $y = 0.25 + 0.5(x - 2)$ linear

Use functions to model relationships between quantities.

Standard
8.F.4 – Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (*x*, *y*) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Explanation: Students identify the rate of change (slope) and initial value (y-intercept) from tables, graphs, equations, or verbal descriptions.

Examples:

- The table below shows the cost of renting a car. The company charges $45 a day for the car as well as charging a one-time, $25 fee for the car's navigation system (GPS). Write an expression for the cost in dollars, c, as a function of the number of days, d.

  Students might write the equation $c = 45d + 25$ using the verbal description or by first making a table.

Days (d)	**Cost (c) in Dollars**
1	70
2	115
3	160
4	205

  Students should recognize that the rate of change is 45 (the cost of renting the car) and that initial cost (the first-day charge) also includes paying for the navigation system. Classroom discussion about one-time fees vs. recurrent fees will help student's model contextual situations.

- When scuba divers come back to the surface of the water, they need to be careful not to ascend too quickly. Divers should not come to the surface more quickly than a rate of 0.75 ft per second. If the divers start at a depth of 100 feet, the equation $d = 0.75t - 100$ shows the relationship between the time of the ascent in seconds (t) and the distance from the surface in feet (d).
  - Will they be at the surface in 5 minutes? How long will it take the divers to surface from their dive?
- Make a table of values showing several times and the corresponding distance of the divers from the surface. Explain what your table shows. How do the values in the table relate to your equation?

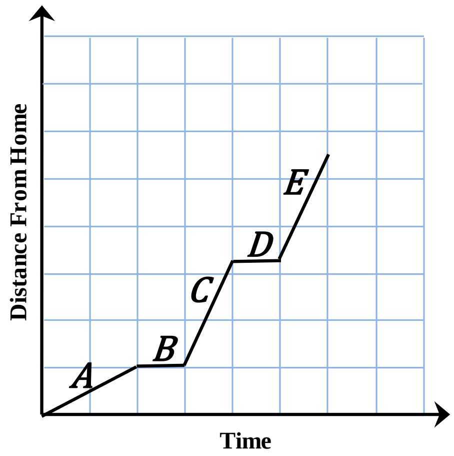
Standard
8.F.5 – Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Explanation: Given a verbal description of a situation, students sketch a graph to model that situation. Given a graph of a situation, students provide a verbal description of the situation.

Example:

- The graph below shows a student's trip to school. This student walks to his friend's house, and together they ride a bus to school. The bus stops once before arriving at school.

  Describe how each Part A– E of the graph relates to the story.



Geometry (G)

Understand congruence and similarity using physical models, transparencies, or geometry software.

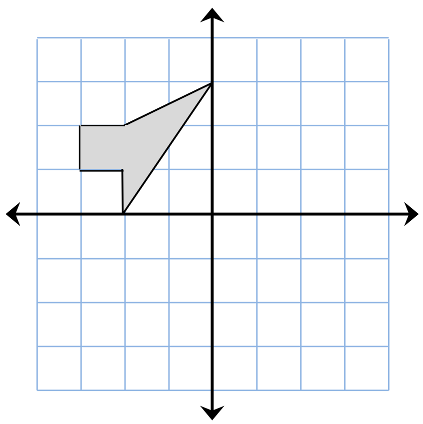
Standard
8.G.1 – Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.

Explanations:

- Students need multiple opportunities to explore the transformation of figures so that they can appreciate that points stay the same distance apart and lines stay at the same angle after they have been rotated, reflected, and/or translated.
- Students are not expected to work formally with properties of dilations until high school.

Examples:

- Aaron is drawing some designs for greeting cards. He divides a grid into 4 quadrants and starts by drawing a shape in one quadrant. He then reflects, rotates, or translates the shape into the other three quadrants.
  - Finish Aaron's first design by reflecting the gray shape over the vertical line. Then, reflect both of the shapes over the horizontal line. This will make a design in all four quadrants.

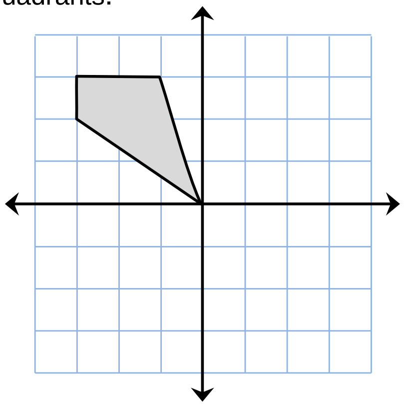


Standard
8.G.1 Examples (continued)

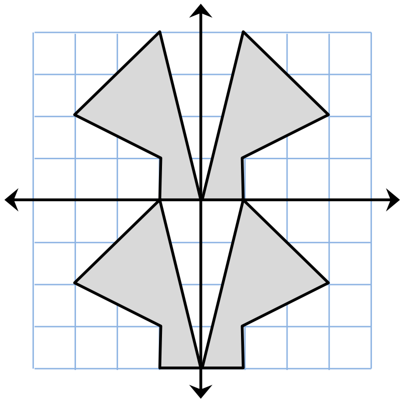
- To finish drawing Aaron's second design, rotate the gray shape $\frac{1}{4}$ of a turn in a clockwise direction about the origin. Then draw the second shape.

  Rotate the second shape $\frac{1}{4}$ of a turn in a clockwise direction about the origin. Then draw the third shape.

  Rotate the third shape $\frac{1}{4}$ of a turn in a clockwise direction about the origin. Then draw the fourth shape. This will make a design in all four quadrants.



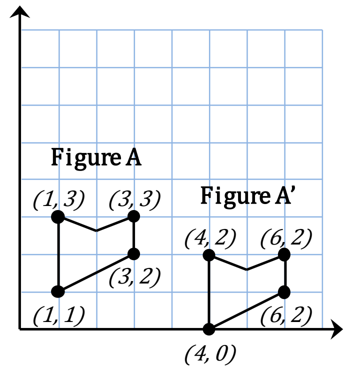
- This is Aaron's third design. He started with one gray shape in the top left-hand quadrant of the grid and transformed it to make the design. Describe the transformations that Aaron may have used to draw this design.



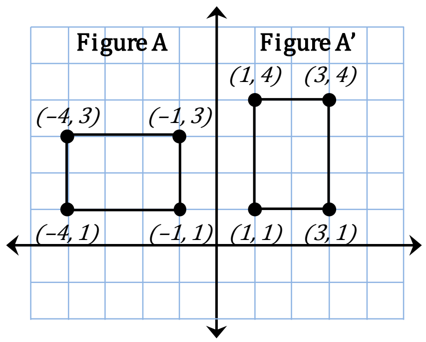
Standard
8.G.2 – Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Explanation: This standard is the students' introduction to congruency. Congruent figures have the same shape and size. Translations, reflections, and rotations are examples of rigid transformations. A rigid transformation is one in which the pre-image and the image both have exactly the same size and shape since the measures of the corresponding angles and corresponding line segments remain equal (are congruent).

- Is Figure A congruent to Figure A'? Explain how you know.



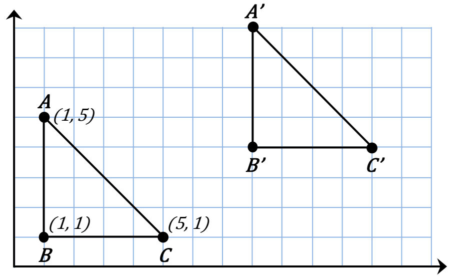
- Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



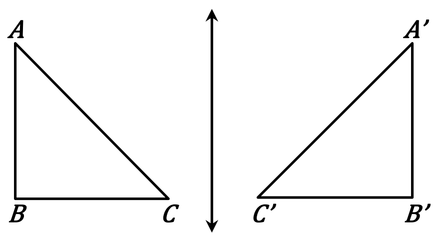
Standard
8.G.3 – Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Explanations:

- A dilation is a transformation that moves each point along a ray starting from a fixed center, and multiplies distances from the center by a common scale factor. In dilated figures, the dilated figure is *similar* to its pre-image.
- Translation: A translation is a transformation of an object that moves the object so that every point of the object moves in the same direction as well as the same distance. In a translation, the translated object is *congruent* to its pre-image. ΔABC has been translated 7 units to the right and 3 units up. To get from A (1,5) to A' (8,8), move A 7 units to the right (from $x = 1$ to $x = 8$) and 3 units up (from $y = 5$ to $y = 8$). Points B and C also move in the same direction (7 units to the right and 3 units up).



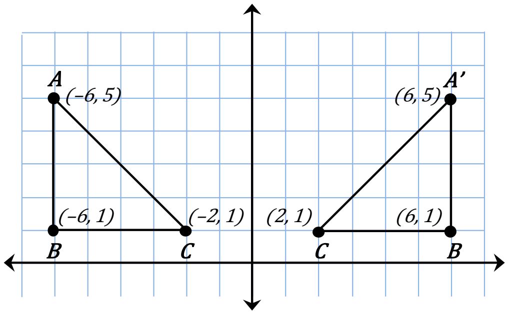
- Reflection: A reflection is a transformation that flips an object across a line of reflection (in a coordinate grid the line of reflection may be the x or y axis). In a rotation, the rotated object is *congruent* to its pre-image.



$$\triangle ABC \cong \triangle A'B'C'$$

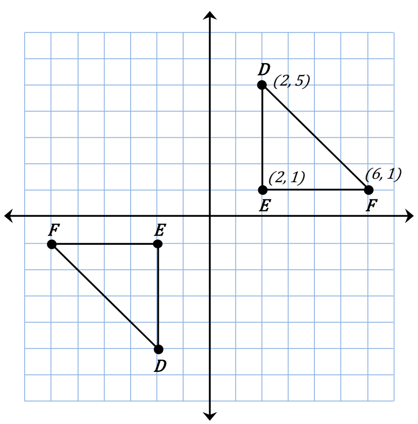
Standard
8.G.3 Explanations (continued)

- When an object is reflected across the y-axis, the reflected x-coordinate is the opposite of the pre-image x-coordinate.



- Rotation: A rotated figure is a figure that has been turned about a fixed point. This is called the center of rotation. A figure can be rotated up to 360°. Rotated figures are congruent to their pre-image figures.

  Consider when ΔDEF is rotated 180° clockwise about the origin. The coordinates of ΔDEF are $D(2,5)$, $E(2,1)$, and $F(6,1)$. When rotated 180°, $\Delta D'E'F'$ has new cooridnates $D'(-2,-5)$, $E'(-2,-1)$, and $F'(-6,-1)$. Each coordinate is the opposite of its pre-image.

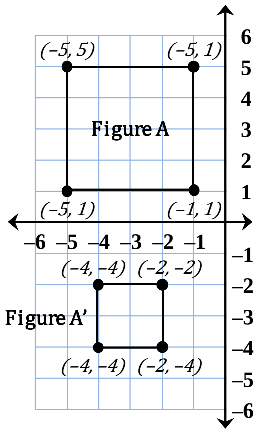


Standard
8.G.4 Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

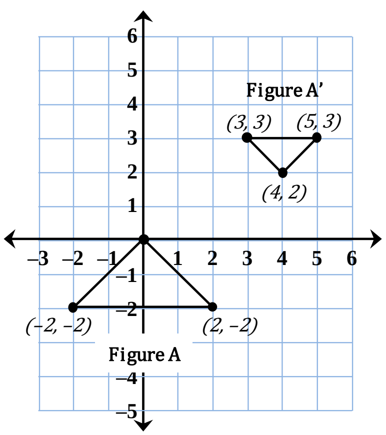
Explanation: This is the students' introduction to similarity and similar figures. Students understand similar figures have angles with the same measure and sides that are proportional. Similar figures are produced from dilations.

Examples:

- Is Figure A similar to Figure A'? Explain how you know.



- Describe the sequence of transformations that results in the transformation of Figure A to Figure A'.



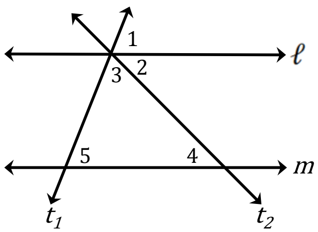
Standard

8.G.5 – Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

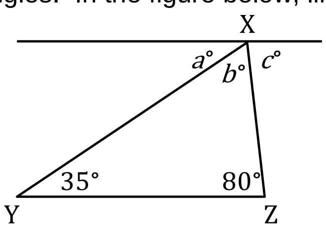
Explanation: Students use exploration and deductive reasoning to determine relationships that exist between a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle.

Examples:

- Students can informally prove relationships with transversals.
- Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180°$ if ℓ and m are parallel lines and t_1 and t_2 are transversals.
  - $\angle 1 + \angle 2 + \angle 3 = 180°$. $\angle 1$ and $\angle 5$ are congruent because they are corresponding angles $(\angle 5 \cong \angle 1)$. $\angle 1$ can be substituted for $\angle 5$.
  - $\angle 4 \cong \angle 2$ because alternate interior angles are congruent.
  - $\angle 4$ can be substituted for $\angle 2$.
  - Therefore, $m\angle 3 + m\angle 4 + m\angle 5 = 180°$.



- Students can conclude that the sum of a triangle is 180° (the angle-sum theorem) by applying their understanding of lines and alternate interior angles. In the figure below, line X is parallel to line YZ:



$\overline{X} \| \overline{YZ}$

Because it alternates with the angle inside the triangle that measures $35°$, $m\angle a$ is $35°$. Because it alternates with the angle inside the triangle that measures 80º, $m\angle c$ is $80°$. Because lines have a measure of $180°$ and $\angle a + \angle b + \angle c$ form a straight line, then $\angle b$ must be $65°$ $(180 - 35 - 80 = 65)$. Therefore, the sum of the angles of the triangle are $35° + 65° + 80°$.

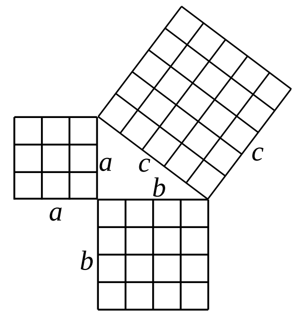
Understand and apply the Pythagorean Theorem.

Standard
8.G.6 – Explain a proof of the Pythagorean Theorem and its converse.

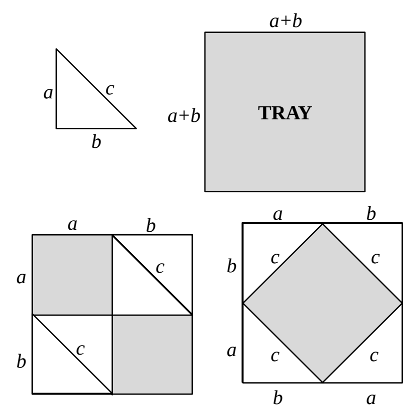
Explanation: Students should verify, using a model, that the sum of the squares of the legs is equal to the square of the hypotenuse in a right triangle. Students should also understand that if the sum of the squares of the 2 smaller legs of a triangle is equal to the square of the third leg, then the triangle is a right triangle. Students also understand that given three side lengths with this relationship forms a right triangle.

Examples: Following are three attempts to prove the Pythagorean theorem. Look carefully at each attempt. Which is the best "proof"? Explain your reasoning as fully as possible.

- ***Attempt 1:*** Suppose a right triangle has sides of length a, b, and c.
  - Draw squares on the three sides as shown.
  - You can see that the number of squares on the two shorter sides add up to make the number of squares on the longest side.
  - So: $a^2 + b^2 = c^2$

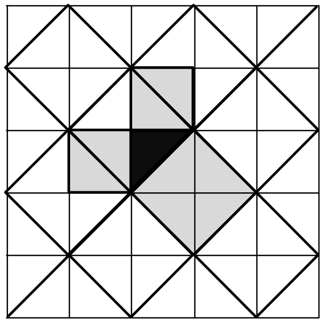


- ***Attempt 2:*** Suppose that you start with four right triangles with sides of length a, b, and c and a square tray with sides of length $a + b$.
  - You can arrange the triangles into the tray in two different ways as shown here. In the first way, you leave two square holes. These have a combined area of $a^2 + b^2$. In the second, way, you leave one large square hole. This has an area of c^2.
  - Since these areas are equal: $a^2 + b^2 = c^2$.



Standard

- ***Attempt 3:*** The proof of the Pythagorean Theorem is clear from this diagram.
  - The squares on the two shorter sides of the black triangle are each made from two congruent triangles.
  - These fit together to make the square on the longest side the hypotenuse.



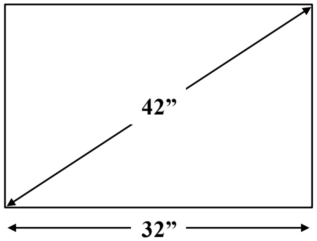
- Provide answers to the following:
  - The best proof number is: _____.
  - This is because:
  - My assessment of the others is:

8.G.7 – Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

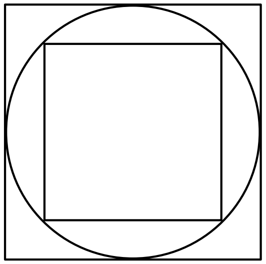
Explanation: Through authentic experiences and exploration, students should use the Pythagorean Theorem to solve problems. Problems can include working in both two and three dimensions. Students should be familiar with the common Pythagorean triplets $(3^2 + 4^2 = 5^2)$.

Examples:

- Jane is hoping to buy a large new television for her den, but she is not sure what size screen will be suitable for her wall. The is because television screens are measured by their diagonal line. The following 42-inch screen measures 32 inches along the base.
  - What is the height of the screen? Show how you know.
  - What is the area of the screen in square inches?
  - Jane would like to have a screen 40-inches wide and 32-inches high. What screen size, in inches, will she need to buy? Show your work.



- The following diagram shows a circle with one square inside and one square outside.
  - What is the ratio of the areas of the two squares? Show your work.
  - If a second circle is inscribed inside the smaller square, what is the ratio of the areas of the two circles? Explain your reasoning.

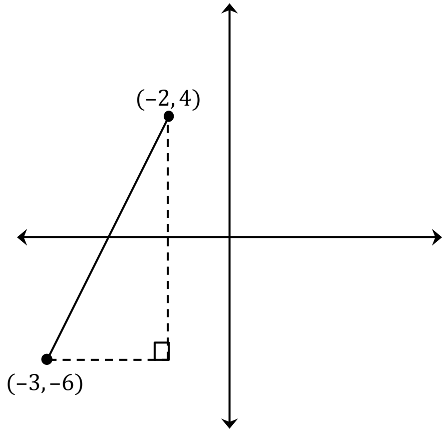


Standard
8.G.8 – Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Explanation: One application of the Pythagorean Theorem is finding the distance between two points on the coordinate plane. Students understand that the line segment between the two points is the length of the hypotenuse.

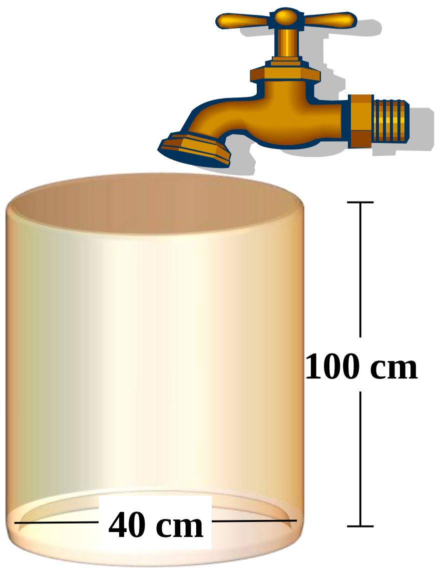
Example:

- Students will create a right triangle from the two points given (as shown in the diagram below) and then use the Pythagorean Theorem to find the distance between the two given points.



Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Standard
8.G.9 – Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems. *Explanation:* Students build on understandings of circles and volume from 7th grade to find the volume of cylinders, cones, and spheres and use them to solve real-world problems. *Example:* • James wanted to fill his container with water. He wondered how much water he would need to fill it. Use the measurements in the diagram below to determine the container's volume.



Statistics and Probability (SP)

Investigate patterns of association in bivariate data.

Standard
8.SP.1 – Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

Explanation: Students build on their previous knowledge of scatter plots to examine relationships between variables. Bivariate data refers to two variable data, one to be graphed on the x-axis and the other on the y-axis. The students analyze scatter plots to determine positive and negative associations, the degree of association, and type of association. Students examine outliers to determine if data points are valid or represent a recording or measurement error. Students can use tools such as those at the National Center for Educational Statistics to create a graph or generate data sets (http://nces.ed.gov/nceskids/createagraph/default.aspx)).

Examples:

- Data for 10 students' math and science scores are provided in the following chart. Describe the association between the math and science scores.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Math	64	50	85	34	56	24	72	63	42	93
Science	68	70	83	33	60	27	74	63	40	96

- Data for 10 students' math scores and the distance they live from school are provided in the table below. Describe the association between the math scores and the distance they live from school.

Student	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
Math	64	50	85	34	56	24	72	63	42	93
Distance from school (miles)	0.5	1.8	1.0	2.3	3.4	0.2	2.5	1.6	0.8	2.5

- Data from a local fast food restaurant is provided showing the number of staff members and the average time for filling an order are provided in the table below. Describe the association between the number of staff and the average time for filling an order.

Number of staff	3	4	5	6	7	8
Average time to fill order (seconds)	180	138	120	108	96	84

- The chart below lists the life expectancy in years for people in the United States every five years from 1970 to 2005. What would you expect the life expectancy of a person in the United States to be in 2010, 2015, and 2020 based upon this data? Explain how you determined your values.

Date	1970	1975	1980	1985	1990	1995	2000	2005
Life Expectancy (in years)	70.8	72.6	73.7	74.7	75.4	75.8	76.8	77.4

Standard
8.SP.2 – Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Example:

- The capacity of the fuel tank in a car is 13.5 gallons. The table below shows the number of miles traveled and how many gallons of gas are left in the tank. Describe the relationship between the variables. If the data is linear, determine a line of best fit. Do you think the line represents a good fit or the data set? Why or why not? What is the average fuel efficiency of the car in miles per gallon?

Miles Traveled	0	75	120	160	250	300
Gallons Used	0	2.3	4.5	5.7	9.7	10.7

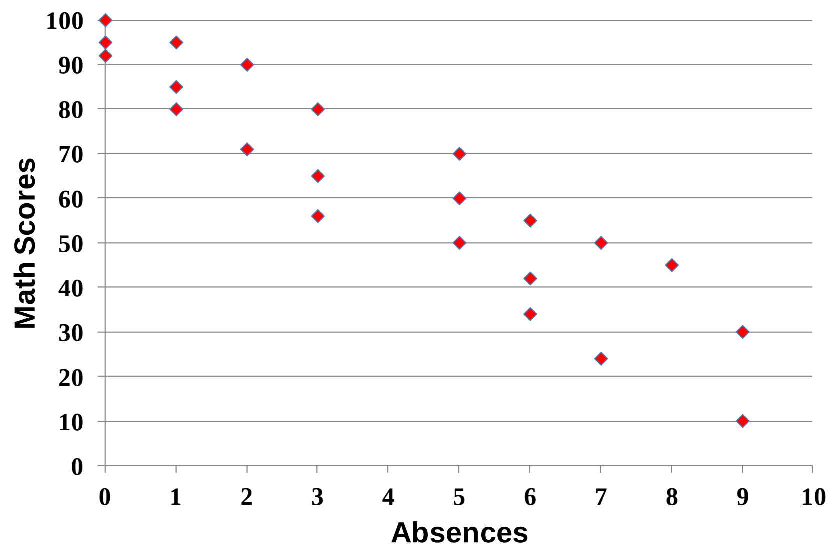
Standard
8.SP.3 – Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height*.

Explanation: Linear models can be represented with a linear equation. Students interpret the slope and y-intercept of the line in the context of the problem.

Examples:

1. Given data from students' math scores and absences, make a scatter plot.

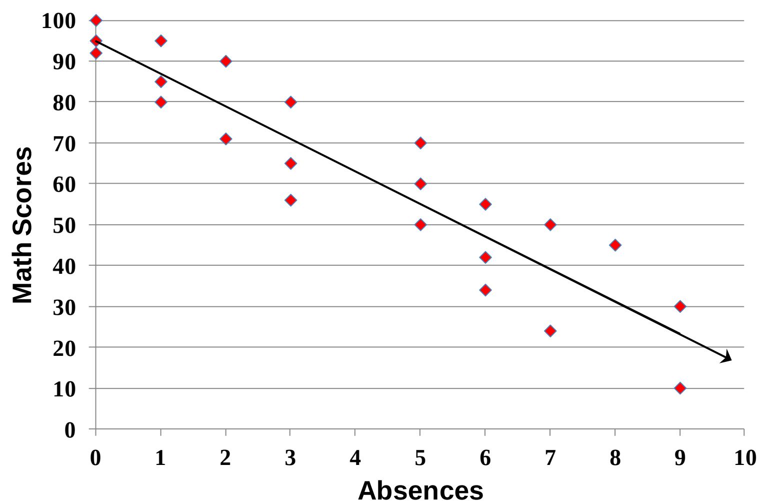
Absences	Math Scores
3	65
5	50
1	95
1	85
3	80
6	34
5	70
3	56
0	100
7	24
8	45
2	71
9	30
0	95
6	55
6	42
2	90
0	92
5	60
7	50
9	10
1	80



Standard

8.SP.3 Examples (continued)

2. Draw a line of best fit, paying attention to the closeness of the data points on either side of the line.



3. From the line of best fit, determine an approximate linear equation that models the given data (about $y = -\frac{25}{3}x + 95$).
4. Students should recognize that 95 represents the y intercept and $-\frac{25}{3}$ represents the slope of the line.
5. Students can use this linear model to solve problems. For example, through substitution, they can use the equation to determine that a student with 4 absences should expect to receive a math score of about 62. They can compare this value to their line.

Standard
8.SP.4 – Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Explanation: Students recognize that categorical data can also be described numerically through the use of a two-way table. A two-way table is a table that shows categorical data classified in two different ways.

Example:

- The table illustrates the results when 100 students were asked the survey questions: Do you have a curfew and do you have assigned chores? Is there evidence that those who have a curfew also tend to have chores?

Curfew

Chores		**Yes**	**No**
	Yes	40	10
	No	10	40

  - Solution: Of the students who answered that they had a curfew, 40 had chores, and 10 did not. Of the students who answered they did not have a curfew, 10 had chores, and 40 did not. From this sample, there appears to be a positive correlation between having a curfew and having chores.