
Delaware's Common Core State Standards for Mathematics Grade 5 Assessment Examples



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Delaware's Common Core State Standards for 5th Grade Mathematics

Overview

Operations and Algebraic Thinking (OA)

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten (NBT)

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions (NF)

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data (MD)

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry (G)

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices (MP)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Grade 5 Mathematics – Unpacking the Delaware Common Core State Standards

This document is designed to help understand the Common Core State Standards (CCSS) in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

What Is the Purpose of This Document?

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

References

This document contains explanations and examples that were obtained from State Departments of Education for Kansas, Utah, Arizona, North Carolina, and Ohio with permission.

How Do I Send Feedback?

This document is helpful in understanding the CCSS but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education via rfry@doe.k12.de.us, and we will use your input to refine this document.

Operations and Algebraic Thinking (OA)

Write and interpret numerical expressions.

Standard

5.OA.1 – Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.

Explanation:

- Begin with expressions that have two operations without any grouping symbols (multiplication or division combined with addition or subtraction) before introducing expressions with multiple operations. Using the same digits, with the operations in a different order, have students evaluate the expressions and discuss why the value of the expression is different. For example, have students evaluate $5 \times 3 + 6$ and $5 + 3 \times 6$. Discuss the rules that must be followed. Have students insert parentheses around the multiplication or division part in an expression. A discussion should focus on the similarities and differences in the problems and the results. This leads to students being able to solve problem situations which require that they know the order in which operations should take place.

After students have evaluated expressions without grouping symbols, present problems with one grouping symbol, beginning with parentheses, then in combination with brackets and/or braces.

- **5.OA.1** calls for students to evaluate expressions with parentheses (), brackets [] and braces { }. In upper levels of mathematics, evaluate means to substitute for a variable and simplify the expression. However, at this level, students are to only simplify the expressions because there are no variables.

Resources:

- Calculators (scientific or four-function)
- NCTM Illuminations: [Order of Operations Bingo](#). Instead of calling numbers to play Bingo, you call (and write) numerical expressions to be evaluated for the numbers on the Bingo cards. The operations in this lesson are addition, subtraction, multiplication, and division; the numbers are all single-digit, whole numbers.

Example:

- Evaluate the expression $2\{5[12 + 5(500 - 100) + 399]\}$
 - Students should have experience working with the order of first evaluating terms in parentheses, then brackets, and then braces.
 - The first step would be to subtract: $500 - 100 = 400$.
 - Then multiply: $400 \times 5 = 2000$.
 - Inside the bracket is: $[12 + 2000 + 399]$. This is equal to 2411.
 - Next, multiply the 5 outside of the bracket: $2411 \times 5 = 12055$.
 - Finally, multiply by the 2 outside of the braces: $12055 \times 2 = 24110$.
 - Mathematically, there cannot be brackets or braces in a problem that does not have parentheses. Likewise, there cannot be braces in a problem that does not have both parentheses and brackets.

Standard

5.OA.2 – Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as $2 \times (8 + 7)$. Recognize that $3 \times (18932 + 921)$ is three times as large as $18932 + 921$, without having to calculate the indicated sum or product.*

Explanation: 5.OA.2 refers to expressions. **Expressions** are a series of numbers and symbols (+, −, ×, ÷) without an equals sign. **Equations** result when two expressions are set equal to each other ($2 + 3 = 4 + 1$).

Example:

- $4(5 + 3)$ is an expression. When we compute $4(5 + 3)$, we are evaluating the expression. The expression equals 32. $4(5 + 3) = 32$ is an equation.
 - 5.OA.2 calls for students to verbally describe the relationship between expressions without actually calculating them.
- Write an expression for the steps “double five and then add 26.”
 - Solution: $(2 \times 5) + 26$

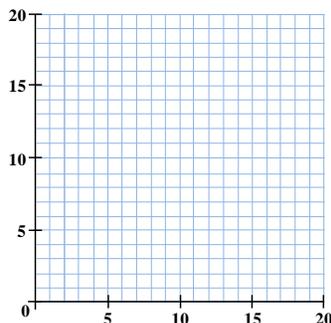
Analyze patterns and relationships.

Standard

5.OA.3 – Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Explanation: 5.OA.3 extends the work from 4th grade, where students generate numerical patterns when they are given one rule. In 5th grade, students are given two rules and generate two numerical patterns.

- Given two rules with an apparent relationship, students should be able to identify the relationship between the resulting sequences of the terms in one sequence to the corresponding terms in the other sequence. For example, starting with 0, multiply by 4 and starting with 0, multiply by 8 and generate each sequence of numbers (0, 4, 8, 12, 16, ...) and (0, 8, 16, 24, 32, ...). Students should see that the terms in the second sequence are double the terms in the first sequence, or that the terms in the first sequence are half the terms in the second sequence.
- Have students form ordered pairs and graph them on a coordinate plane. Patterns can be also discerned in graphs.
 - Graphing ordered pairs on a coordinate plane is introduced to students in the Geometry domain where students solve real-world and mathematical problems. For the purpose of this cluster, only use the first quadrant of the coordinate plane, which contains positive numbers only. Provide coordinate grids for the students but also have them make coordinate grids. In Grade 6, students will position pairs of integers on a coordinate plane.



- The graph of both sequences of numbers is a visual representation that will show the relationship between the two sequences of numbers.
- Have students represent the sequences in the T-chart so that they can see a connection between the graph and the sequences.

0	0
1	4
2	8
3	12
4	16

0	0
1	8
2	16
3	24
4	32

Number and Operations in Base Ten (NBT)

Understand the place value system.

Standard
<p>5.NBT.1 – Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.</p> <p><i>Examples:</i></p> <ul style="list-style-type: none">• Students might write:<ul style="list-style-type: none">▪ $36 \times 10 = 36 \times 10^1 = 360$▪ $36 \times 10 \times 10 = 36 \times 10^2 = 3600$▪ $36 \times 10 \times 10 \times 10 = 36 \times 10^3 = 36,000$• Students might say:<ul style="list-style-type: none">▪ I noticed that every time I multiplied by 10, I added a zero to the end of the number. That makes sense because each digit's value became 10 times larger. To make a digit 10 times larger, I have to move it one place value to the left.▪ When I multiplied 36 by 10, the 30 became 300. The 6 became 60, or the 36 became 360. So I had to add a 0 at the end to have the 3 represent 3 one-hundreds (instead of 3 tens) and the 6 represents 6 tens (instead of 6 ones).• Students should be able to use the same type of reasoning as above to explain why the following multiplication and division problem by powers of 10 make sense.<ul style="list-style-type: none">▪ $523 \times 10^3 = 523,000$ – The place value 523 is increased by 3 places.▪ $5.223 \times 10^2 = 522.3$ – The place value of 5.223 is increased by 2 places.▪ $52.3 \div 10^1 = 5.23$ - The place value of 52.3 is decreased by 1 place.
<p>5.NBT.2 – Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.</p> <p><i>Explanation:</i> They use concrete models and number lines to extend this understanding to decimals to the thousandths. Models may include base ten blocks, place value charts, grids, pictures, drawings, manipulatives, technology-based, etc. They read decimals using fractional language and write decimals in fractional form as well as in expanded notation as show in the standard 3a. This investigation leads them to understanding equivalence of decimals ($0.8 = 0.80 = 0.800$).</p>

Standard

5.NBT.2 Examples:

- Some equivalent forms of 0.72 are:

$\frac{72}{100}$	$\frac{70}{100} + \frac{2}{100}$
$\frac{7}{10} + \frac{2}{100}$	0.720
$7 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right)$	$7 \times \left(\frac{1}{10}\right) + 2 \times \left(\frac{1}{100}\right) + 0 \times \left(\frac{1}{1000}\right)$
$0.70 + 0.02$	$\frac{720}{1000}$

- Students need to understand the size of decimal numbers and relate them to common benchmarks such as 0, 0.5 (0.50 and 0.500), and 1. Comparing tenths to tenths, hundredths to hundredths, and thousandths to thousandths is simplified if students use their understanding of fractions to compare decimals.
 - Comparing 0.25 and 0.17, a student might think, “25 hundredths is more than 17 hundredths.” They may also think that it is 8 hundredths more. They may write this comparison as $0.25 > 0.17$ and recognize that $0.17 < 0.25$ is another way to express this comparison.
 - Comparing 0.207 to 0.26, a student might think, “Both numbers have 2 tenths, so I need to compare the hundredths. The second number has 6 hundredths and the first number has no hundredths, so the second number must be larger.” Another student might think while writing fractions, “I know that 0.207 is 207 thousandths (and may write $\frac{207}{1000}$). 0.26 is 26 hundredths (and may write $\frac{26}{100}$), but I can also think of it as 260 thousandths ($\frac{260}{1000}$). So, 260 thousandths is more than 207 thousandths.”

Standard

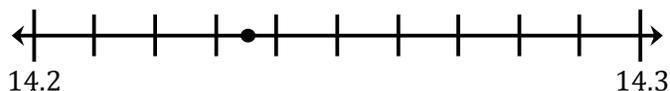
5.NBT.3 – Read, write, and compare decimals to thousandths.

- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
- Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons.

Explanation: When rounding a decimal to a given place, students may identify the two possible answers and use their understanding of place value to compare the given number to the possible answers.

Example:

- Round 14.235 to the nearest tenth. Students recognize that the possible answer must be in tenths, thus it is either 14.2 or 14.3. They then identify that 14.235 is closer to 14.2 (14.20) than to 14.3 (14.30).

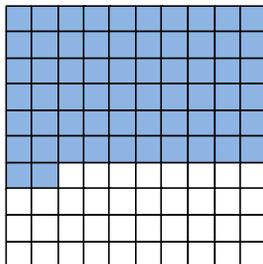


5.NBT.4 – Use place value understanding to round decimals to any place.

Explanation: Students should go beyond simply applying an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line to support their work with rounding.

Example:

- Students should use benchmark numbers to support this work. Benchmarks are convenient numbers for comparing and rounding numbers. Examples of benchmark numbers are: 0, 0.5, 1, 1.5. Which benchmark number is the best estimate of the shaded amount in the model below? Explain your thinking.



Perform operations with multi-digit whole numbers and with decimals to hundredths.

Standard
<p>5.NBT.5 – Fluently multiply multi-digit whole numbers using the standard algorithm.</p> <p><i>Explanation:</i> In prior grades, students used various strategies to multiply. Students can continue to use these different strategies as long as they are efficient, but must also understand and be able to use the standard algorithm. In applying the standard algorithm, students recognize the importance of place value.</p> <p><i>Example:</i></p> <ul style="list-style-type: none"> • 123×34 – When students apply the standard algorithm, they decompose 34 into $30 \div 4$. Then, they multiply 123 by 4, the value of the number in the ones place, and then multiply 123 by 30, the value of the 3 in the tens place. Finally, they add the two products. <ul style="list-style-type: none"> ▪ You can multiply by listing all the partial products. For example: <div style="margin-left: 20px;"> $\begin{array}{r} 234 \\ \times 8 \\ \hline 32 \\ 240 \\ \underline{1600} \\ 1872 \end{array}$ <p>Multiply the ones (8×4 ones = 32 ones) Multiply the tens (8×3 tens = 24 tens or 240) Multiply the hundreds (8×2 hundreds = 16 hundreds or 1600) Add the partial products</p> $\begin{array}{r} 234 \\ \times 8 \\ \hline 1872 \end{array}$ </div> <p>Students should learn to estimate decimal computations before they compute with pencil and paper. The focus on estimation should be on the meaning of the numbers and the operations—not on how many decimal places are involved.</p> <p><i>Resources:</i> Decimal Place Value Chart from the National Library of Virtual Manipulatives</p>

Standard

5.NBT.6 – Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Explanation: In 4th grade, students' experiences with division were limited to dividing by one-digit divisors. This standard extends students' prior experiences with strategies, illustrations, and explanations. When the two-digit divisor is a "familiar" number, a student might decompose the dividend using place value.

Examples:

- Using expanded notation $2682 \div 25 = (2000 + 600 + 80 + 2) \div 25$
 - Using his or her understanding of the relationship between 100 and 25, a student might think:
 - ♦ I know that 100 divided by 25 is 4, so 200 divided by 25 is 8, and 2000 divided by 25 is 80.
 - ♦ 600 divided by 25 has to be 24.
 - ♦ Since $3 \times 25 = 75$, I know that 80 divided by 25 is 3 with a remainder of 5. (Note that a student might divided into 82 and not 80.)
 - ♦ I cannot divide 2 by 25 so 2 plus the 5 leaves a remainder of 7.
 - ♦ $80 + 24 + 3 = 107$ so the answer is 107 with a remainder of 7.

Using an equation that relates division to multiplication, $25 \times n = 2682$, a student might estimate the answer to be slightly larger than 100 because he or she recognizes that $25 \times 100 = 2500$.

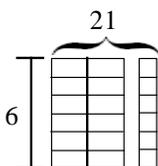
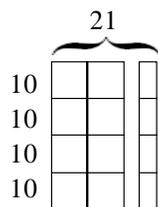
- $968 \div 21$
 - Using base ten models, a student can represent 966 and use the models to make an array with one dimension of 21. The student continues to make the array until no more groups of 21 can be made. Remainders are not part of the array.

So, $210 \times 4 = 840$

$21 \times 6 = \underline{126}$
966

+2 remainder

So, $(21 \times 46) + 2 = 968$



Standard

5.NBT.6 Examples (continued)

- $9984 \div 64$
 - An area model for division is shown below. As students use the area model, they keep track of how much of the 9984 is left to divide.

	64	
100	6400	$64 \overline{)9984}$
	3200	$\underline{-6400}$ 100×64
50	320	3584
	320	$\underline{-3200}$ 50×64
5	64	384
	64	$\underline{-320}$ 5×64
1	64	64
		$\underline{-64}$ 1×64
		0

So, $9984 \div 64 = 156$.

5.NBT.7 – Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Explanation: This standard requires students to extend the models and strategies they developed for whole numbers in grades 1–4 to decimal values. Before students are asked to give exact answers, they should estimate answers based on their understanding of operations and the value of the numbers.

Examples:

- $3.6 + 1.7$ – A student might estimate the sum to be larger than 5 because 3.6 is more than $3\frac{1}{2}$ and 1.7 is more than $1\frac{1}{2}$.
- $5.4 - 0.8$ – A student might estimate the answer to be a little more than 4.4 because a number less than 1 is being subtracted.
- 6×2.4 – A student might estimate an answer between 12 and 18 since $6 \times 2 = 12$ and $6 \times 3 = 18$. Another student might give an estimate of a little less than 15 because he/she figures the answer to be very close but small than $6 \times 2\frac{1}{2}$ and think of $2\frac{1}{2}$ groups of 6 as 12 (2 groups of 6) + 3 ($\frac{1}{2}$ of a group of 6).

Students should be able to express that, when they add decimals, they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in 4th grade.

Standard

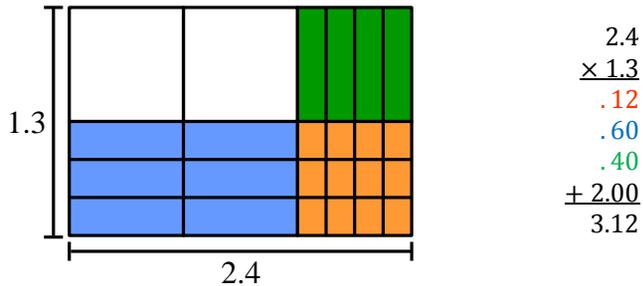
5.NBT.7 Examples (continued)

- $4 - 0.3$ means 3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.



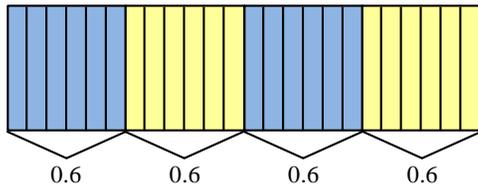
The answer is $3 \frac{7}{10}$ or 3.7.

- An area model can be useful for illustrating products.



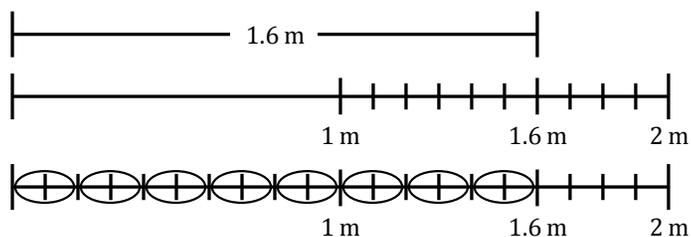
Students should be able to describe the partial products displayed by the area model. For example:

- $\frac{3}{10} \times \frac{4}{10} = \frac{12}{100}$
- $\frac{3}{10} \times 2 = \frac{6}{10}$ or $\frac{60}{100}$
- $1 \times \frac{4}{10} = \frac{4}{10}$ or $\frac{40}{100}$
- $1 \times 2 = 2$
- Find the number in each group or share (division):
 - Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as $2.4 \div 4 = 0.6$.



Standard

- Find the number of groups (division):
 - Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many pieces can he cut? To divide to find the number of groups, a student might:
 - ♦ Draw a segment to represent 1.6 meters. In doing so, he/she would count in tenths to identify the 6 tenths, and be able to identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the 1 meter into tenths and determine that there are 5 more groups of 2 tenths.



- ♦ Count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as $\frac{10}{10}$, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths ... 16 tenths, a student can count 8 groups of 2 tenths.
- ♦ Use their understanding of multiplication and think, "8 groups of 2 is 16, so 8 groups of $\frac{2}{10}$ is $\frac{16}{10}$ or $1\frac{6}{10}$."

Number and Operations—Fractions (NF)

Use equivalent fractions as a strategy to add and subtract fractions.

Standard

5.NF.1 – Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*

Explanation: Students should apply their understanding of equivalent fractions developed in 4th grade and their ability to rewrite fractions in an equivalent form to find common denominators. They should know that multiplying the denominators will always give a common denominator but may not result in the smallest denominator.

Examples:

- $\frac{2}{5} + \frac{7}{8} = \frac{16}{40} + \frac{35}{40} = \frac{51}{40}$
- $3\frac{1}{4} - \frac{1}{6} = 3\frac{3}{12} - \frac{2}{12} = 3\frac{1}{12}$

Instructional Strategies:

- To add or subtract fractions with unlike denominators, students use their understanding of equivalent fractions to create fractions with the same denominators. Start with problems that require the changing of one of the fractions and progress to changing both fractions. Allow students to add and subtract fractions using different strategies such as number lines, area models, fraction bars, or strips. Have students share their strategies and discuss commonalities in them.
- Students need to develop the understanding that when adding or subtracting fractions, the fractions must refer to the same whole. Any models used must refer to the same whole. Students may find that a circular model might not be the best model when adding or subtracting fractions.
- As with solving word problems with whole number operations, regularly present word problems involving addition or subtraction of fractions. The concept of adding or subtracting fractions with unlike denominators will develop through solving problems. Mental computations and estimation strategies should be used to determine the reasonableness of answers. Students need to prove or disprove whether an answer provided for a problem is reasonable.
- Estimation is about getting useful answers, it is not about getting the right answer. It is important for students to learn which strategy to use for estimation. Students need to think about what might be a close answer.

Resources:

- Links to National Library of Virtual Manipulatives
 - [Fraction Bars](#) – Learn about fractions using fraction bars.
 - [Fractions–Adding](#) – Illustrates what it means to find a common denominator and combine.
 - [Number Line Bars](#) – Use bars to show addition, subtraction, multiplication, and division on a number line.

Standard

5.NF.2 – Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

Explanation: This standard refers to number sense, which means students' understanding of fractions as numbers that lie between whole numbers on a number line. Number sense in fractions also includes moving between decimals and fractions to find equivalents as well as being able to use reasoning such as $\frac{7}{8}$ is greater than $\frac{3}{4}$ because $\frac{7}{8}$ is missing only $\frac{1}{8}$ and $\frac{3}{4}$ is missing $\frac{1}{4}$ so $\frac{7}{8}$ is closer to a whole. Also, students should use benchmark fractions to estimate and examine the reasonableness of their answers. Example: $\frac{5}{8}$ is greater than $\frac{6}{10}$ because $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$ (same as $\frac{4}{8}$), and $\frac{6}{10}$ is only $\frac{1}{10}$ larger than $\frac{1}{2}$ (same as $\frac{5}{10}$).

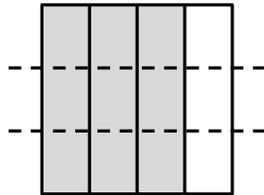
Examples:

- Your teacher gave you $\frac{1}{7}$ of the bag of candy. She also gave your friend $\frac{1}{3}$ of the bag of candy. If you and your friend combined your candy, what fraction of the bag would you have? Estimate your answer and then calculate. How reasonable was your estimate?
 - Solution 1: $\frac{1}{7}$ is really close to 0. $\frac{1}{3}$ is larger than $\frac{1}{7}$ but still less than $\frac{1}{2}$. If we put them together, we might get close to $\frac{1}{2}$.
 - ♦ $\frac{1}{7} + \frac{1}{3} = \frac{3}{21} + \frac{7}{21} = \frac{10}{21}$. The fraction does not simplify. We know that 10 is half of 20, so $\frac{10}{21}$ is less than $\frac{1}{2}$.
 - Solution 2: $\frac{1}{7}$ is close to $\frac{1}{6}$, and $\frac{1}{3}$ is equivalent to $\frac{2}{6}$, so we have a little less than $\frac{3}{6}$ or $\frac{1}{2}$.

Standard

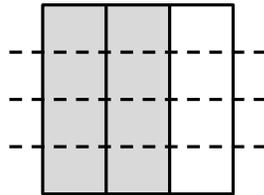
5.NF.2 (continued)

- Jerry was making two different types of cookies. One recipe needed $\frac{3}{4}$ cup of sugar and the other needed $\frac{2}{3}$ cup of sugar. How much sugar did he need to make both recipes?
 - Mental estimation: A student may say that Jerry needs more than 1 cup of sugar but less than 2 cups. An explanation may compare both fractions to $\frac{1}{2}$ and state that both are larger than $\frac{1}{2}$ so the total must be more than 1. Also, both fractions are slightly less than 1 so the sum cannot be more than 2.
 - Area model:



$\frac{3}{4}$ cup of sugar

$$\frac{3}{4} = \frac{9}{12}$$

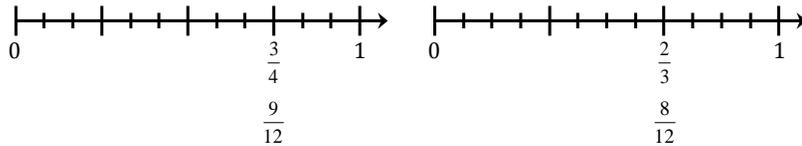


$\frac{2}{3}$ cup of sugar

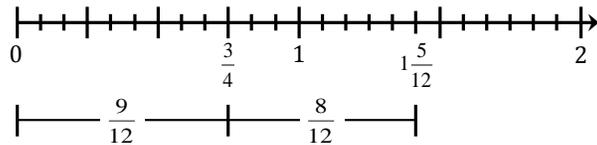
$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{3}{4} + \frac{2}{3} = \frac{17}{12} = \frac{12}{12} + \frac{5}{12} = 1\frac{5}{12}$$

- Linear model:



- Solution:



Standard

5.NF.2 (continued)

• Bar diagram:

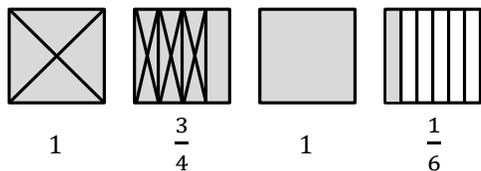
- Sonia had $2\frac{1}{3}$ candy bars. She promised her brother that she would give him $\frac{1}{2}$ of a candy bar. How much will she have left after she gives her brother the amount she promised?
- If Mary ran $3\frac{1}{6}$ miles every week for 4 weeks, she would reach her goal for the month. The first day of the first week she ran $1\frac{3}{4}$ miles. How many more miles does she still need to run the first week?

♦ Using addition: $1\frac{3}{4} + n = 3$.

♦ A student might add $1\frac{1}{4}$ to $1\frac{3}{4}$ to get to 3 miles. Then, he/she would add $\frac{1}{6}$ more. Thus, $1\frac{1}{4}$ miles + $\frac{1}{6}$ of a mile is what Mary needs to run during that week.

• Area model for subtraction:

- This model shows $1\frac{3}{4}$ subtracted from $3\frac{1}{6}$ leaving $1 + \frac{1}{4} + \frac{1}{6}$ which a student can then change to $1 + \frac{3}{12} + \frac{2}{12} = 1\frac{5}{12}$.



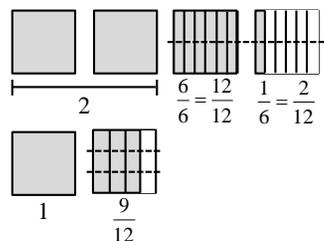
$3\frac{1}{6}$ and $1\frac{3}{4}$ can be expressed with a denominator of 12. Once this is done, a student can complete the problem: $2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$.

Standard

5.NF.2 (continued)

- The following diagram models a way to show how $3\frac{1}{6}$ and $1\frac{3}{4}$ can be expressed with a denominator of 12. Once this is accomplished, a student can

complete the problem: $2\frac{14}{12} - 1\frac{9}{12} = 1\frac{5}{12}$.



- Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies for calculations with fractions extend from students' work with whole number operations and can be supported through the use of physical models.

- Ellie drank $\frac{3}{5}$ quart of milk and Javier drank $\frac{1}{10}$ of a quart less than Ellie. How much milk did they drink all together?

♦ Solution:

$$\frac{3}{5} - \frac{1}{10} = \frac{6}{10} - \frac{1}{10} = \frac{5}{10} \quad \text{This is how much milk Javier drank.}$$

$$\frac{3}{5} + \frac{5}{10} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10} \quad \text{Together they drank } 1\frac{1}{10} \text{ quarts of milk.}$$

This solution is reasonable because Ellie drank more than $\frac{1}{2}$ quart and Javier drank $\frac{1}{10}$ quart, so together they drank slightly more than one quart.

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Standard
<p>5.NF.3 – Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <i>For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</i></p> <p><i>Instructional Strategies:</i> Connect the meaning of multiplication and division of fractions with whole-number multiplication and division. Consider area models of multiplication and both sharing and measuring models for division.</p> <p><i>Instructional Resources:</i></p> <ul style="list-style-type: none"> National Library of Virtual Manipulatives – contains Java applets and activities for grades K–12 mathematics, including Fractions-Rectangle Multiplication and Number Line Bars. <p><i>Explanation:</i> Students are expected to demonstrate their understanding using concrete materials, drawing models, and explaining their thinking when working with fractions in multiple contexts. They read $\frac{3}{5}$ as “three fifths” and after many experiences with sharing problems, learn that $\frac{3}{5}$ can also be interpreted as “3 divided by 5.”</p> <p><i>Examples:</i></p> <ul style="list-style-type: none"> Ten team members are sharing 3 boxes of cookies. How much of a box will each student get? <ul style="list-style-type: none"> When working this problem, a student should recognize that the 3 boxes are being divided into 10 groups, so he or she is seeing the solution to the following equation: $10 \times n = 3$ (10 groups of some amount is 3 boxes), which can also be written as $n = 3 \div 10$. Using models or diagrams, they divide each box into 10 groups, resulting in each team member getting $\frac{3}{10}$ of a box. Two afterschool clubs are having pizza parties. For the Math Club, the teacher will order 3 pizzas for every 5 students. For the student council, the teacher will order 5 pizzas for every 8 students. Since you are in both groups, you need to decide which party to attend. How much pizza would you get at each party? If you want to have the most pizza, which party should you attend? The six 5th grade classrooms have a total of 27 boxes of pencils. How many boxes will each classroom receive? <ul style="list-style-type: none"> Students may recognize this as a whole number division problem but should also express this equal sharing problem as $\frac{27}{6}$. They explain that each classroom gets $\frac{27}{6}$ boxes of pencils and can further determine that each classroom get $4\frac{3}{6}$ or $4\frac{1}{2}$ boxes of pencils.

Standard

5.NF.4 – Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

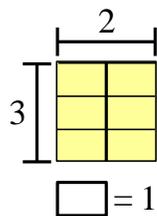
- Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. For example, use a visual fraction model to show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)
- Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

Explanations:

- Students are expected to multiply fractions including proper fractions, improper fractions, and mixed numbers. They multiply fractions efficiently and accurately as well as solve problems in both contextual and non-contextual situations.
- As they multiply fractions such as $\frac{3}{5} \times 6$, they can think of the operation in more than one way.
 - $3 \times (6 \div 5)$ or $(3 \times \frac{6}{5})$
 - $(3 \times 6) \div 5$ or $18 \div 5$ ($\frac{18}{5}$)
- Students create a story problem for $\frac{3}{5} \times 6$, such as:
 - Isabel had 6 feet of wrapping paper. She used $\frac{3}{5}$ of the paper to wrap some presents. How much does she have left?
 - Every day Tim ran $\frac{3}{5}$ of a mile. How far did he run after 6 days? (Interpreting this as $6 \times \frac{3}{5}$)

Examples: Building on previous understandings of multiplication

- Rectangle with dimensions of 2 and 3 showing that $2 \times 3 = 6$.



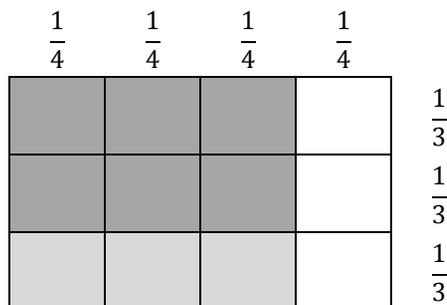
Standard

5.NF.4 (continued)

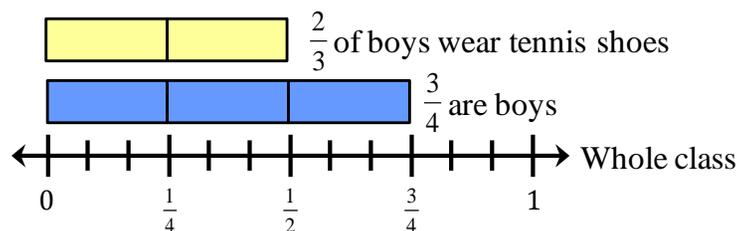
5.NF.4a references both the multiplication of a fraction by a whole number and the multiplication of two fractions. Visual fraction models (area models, tape diagrams, number lines) should be used and created by students during their work with this standard.

Example:

- Three-fourths of the class is boys. Two-thirds of the boys are wearing tennis shoes. What fraction of the class are boys with tennis shoes?
 - This question is asking what $\frac{2}{3}$ of $\frac{3}{4}$ is, or what is $\frac{2}{3} \times \frac{3}{4}$. What is $\frac{2}{3} \times \frac{3}{4}$, in this case you have $\frac{2}{3}$ groups of size $\frac{3}{4}$ (a way to think about it in terms of the language for whole numbers is 4×5 , you have 4 groups of size 5). The array model is very transferable from whole number work and then to binomials.
 - Solutions:
 - ♦ Student 1: I drew a rectangle to represent the whole class. The four columns represent the fourths of a class. I shaded 3 columns to represent the fraction that are boys. I then split the rectangle with horizontal lines into thirds to represent tennis shoes. The dark area represents the fraction of the boys in the class wearing tennis shoes, which is 6 out of 12. That is $\frac{6}{12}$, which equals $\frac{1}{2}$.



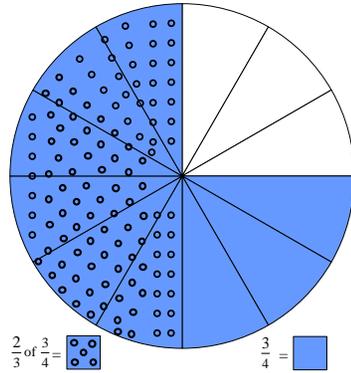
- Student 2:



Standard

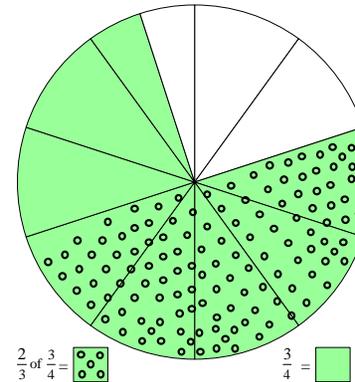
5.NF.4 (continued)

- Student 3: A Fraction Circle could be used to model student thinking – First, I shade the fraction circle to show the $\frac{3}{4}$ and then overlay with $\frac{2}{3}$ of that region.



Using 12s, $\frac{3}{4} = \frac{9}{12}$ and $\frac{2}{3} = \frac{6}{9}$.

Therefore, $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{2}$, as depicted in the double-shaded region.

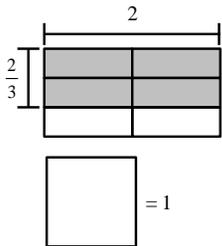


Using 10s, each full section equals $\frac{1}{10}$.

So, $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$ or $\frac{3}{4} = 7 \frac{1}{2}$ sections.

$\frac{2}{3} = \frac{10}{15}$ Therefore, $\frac{2}{3}$ of $\frac{3}{4} = \frac{5}{10}$ or $\frac{1}{2}$ as depicted in the double-shaded region.

- Rectangle with dimensions of 2 and $\frac{2}{3}$ showing that $2 \times \frac{2}{3} = \frac{4}{3}$.

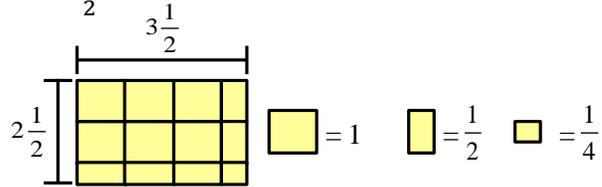


or $\frac{2}{3} + \frac{2}{3} = \frac{4}{3}$ or $4 \times \frac{1}{3} = \frac{4}{3}$

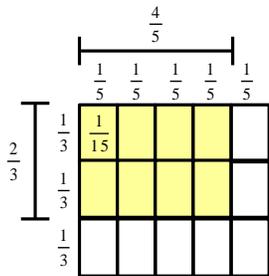
Standard

5.NF.4 (continued)

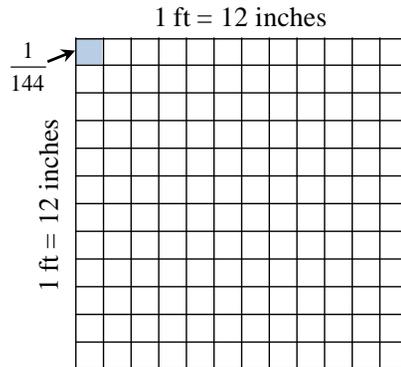
- $2\frac{1}{2}$ groups of $3\frac{1}{2}$



- The area model and the line segments show that the area is the same quantity as the product of the side lengths.



- Larry knows that $\frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$. To prove this he makes the following array.



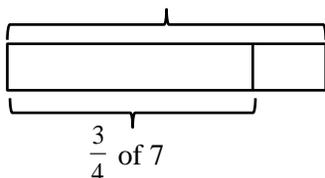
Standard

5.NF.5 – Interpret multiplication as scaling (resizing), by:

- Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
- Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

Examples:

- $\frac{3}{4} \times 7$ is less than 7 because 7 is multiplied by a factor less than 1 so the product must be less than 7.



- $2\frac{2}{3} \times 8$ must be more than 8 because 2 groups of 8 is 16 and $2\frac{2}{3}$ is almost 3 groups of 8. So, the answer must be close to but less than 24.
- $\frac{3}{4} = \frac{5 \times 3}{5 \times 4}$ because multiplying $\frac{3}{4}$ by $\frac{5}{5}$ is the same as multiplying by 1.

Standard

Explanation:

- 5.NF.5b asks students to examine how numbers change when we multiply by fractions. Students should have ample opportunities to examine both cases in the standard: a) when multiplying by a fraction greater than 1, the number increases and b) when multiplying by a fraction less than one, the number decreases. This standard should be explored and discussed while students are working with 5.NF.4, and should not be taught in isolation.

Example:

- Mrs. Bennett is planting two flowerbeds. The first flowerbed is 5 meters long and $\frac{6}{5}$ meters wide. The second flowerbed is 5 meters long and $\frac{5}{6}$ meters wide. How do the areas of these two flowerbeds compare? Is the value of the area larger or smaller than 5 square meters? Draw pictures to prove your answer.

- Evan bought 6 roses for his mother. $\frac{2}{3}$ of them were red. How many red roses were there?

- Using a visual, a student divides the 6 roses into 3 groups and counts how many are in 2 of the 3 groups.



- A student can use an equation to solve: $\frac{2}{3} \times 6 = \frac{12}{3} = 4$ red roses
- Mary and Joe determined that the dimensions of their school flag needed to be $1\frac{1}{3}$ ft. by $2\frac{1}{4}$ ft. What will be the area of the school flag?
 - A student can draw an array to find this product and can also use his/her understanding of decomposing numbers to explain the multiplication. Thinking ahead, a student may decide to multiply by $1\frac{1}{3}$ instead of $2\frac{1}{4}$. The explanation may include one of the following:

Solution 1:

First, I am going to multiply $2\frac{1}{4}$ by 1 and then by $\frac{1}{3}$.

When I multiply $2\frac{1}{4}$ by 1, it equals $2\frac{1}{4}$.

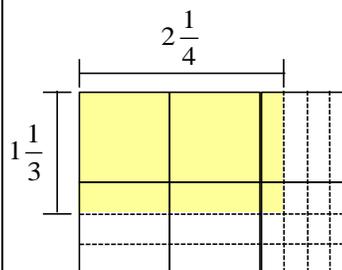
Now I have to multiply $2\frac{1}{4}$ by $\frac{1}{3}$.

$$\frac{1}{3} \times 2 = \frac{2}{3}$$

$$\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$$

So, the answer is $2\frac{1}{4} + \frac{2}{3} + \frac{1}{12}$ or $2\frac{3}{12} + \frac{8}{12} + \frac{1}{12} = 2\frac{12}{12} = 3$

Solution 2:



$$1 + 1 + \frac{1}{3} + \frac{1}{3} + \frac{1}{4} + \frac{1}{12}$$

$$2 + \frac{4}{12} + \frac{4}{12} + \frac{3}{12} + \frac{1}{12}$$

$$2 + \frac{12}{12} = 2 + 1 = 3$$

Standard

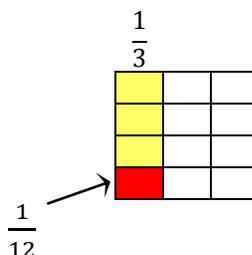
5.NF.7 – Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.*
- Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*
- Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?*

Explanation: In 5th grade, students experience division problems with whole number divisors and unit fraction dividends (fractions with a numerator of 1) or with unit fraction divisors and whole number dividends. Students extend their understanding of the meaning of fractions, how many unit fractions are in a whole, and their understanding of multiplication and division as involving equal groups or shares and the number of objects in each group/share. In 6th grade, they will use this foundational understanding to divide into and by more complex fractions and develop abstract methods of dividing by fractions.

Example (knowing the number of groups/shares and finding how many/much in each group/share):

- Four students sitting at a table were given $\frac{1}{3}$ of a pan of brownies to share. How much of a pan will each student get if they share the pan of brownies equally?
 - The diagram shows the $\frac{1}{3}$ pan divided into 4 equal shares with each share equaling $\frac{1}{12}$ of the pan.



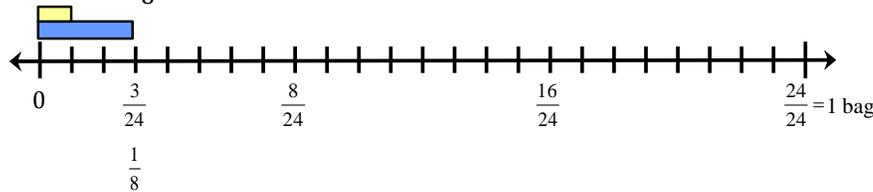
Standard

Explanation for 5.NF.7a: Asks students to work with story contexts where a unit fraction is divided by a non-zero whole number. Students should use various fraction models and reasoning about fractions.

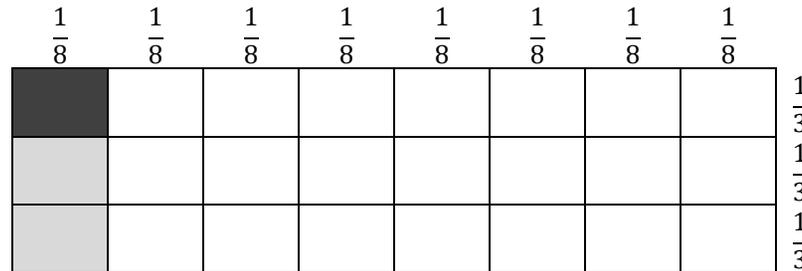
Example 5.NF.7a:

- You have $\frac{1}{8}$ of a bag of pens and you need to share them among 3 people. How much of the bag does each person get?

- Solution 1: $\frac{1}{8} \div 3$



- Solution 2: I drew a rectangle and divided it into 8 columns to represent my $\frac{1}{8}$. I shaded the first column. I then needed to divide the shaded region into 3 parts to represent sharing among 3 people. I shaded one-third of the first column even darker. The dark shade is $\frac{1}{24}$ of the grid or $\frac{1}{24}$ of the bag of pens.



- Solution 3: $\frac{1}{8}$ of a bag of pens divided by 3 people. I know that my answer is less than $\frac{1}{8}$ since I am sharing $\frac{1}{8}$ into 3 groups. I multiply 8×3 , which is 24, so my answer is $\frac{1}{24}$ of the bag of pens. This answer is correct because $\frac{1}{24} \times 3 = \frac{3}{24}$, which equals $\frac{1}{8}$.

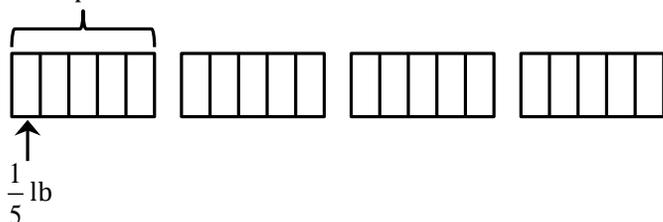
Standard

5.NF.7c Examples:

Knowing how many in each group/share and finding how many groups/shares.

- Angelo has 4 lbs of peanuts. He wants to give each of his friends $\frac{1}{5}$ lb. How many friends can receive $\frac{1}{5}$ lb of peanuts?
 - A diagram for $4 \div \frac{1}{5}$ is shown. Students explain that since there are 5 fifths in one whole, there must be 20 fifths in 4 lbs.

1 lb of peanuts



- How much rice will each person get if 3 people share $\frac{1}{2}$ lb of rice equally?
 - $\frac{1}{2} \div 3 = \frac{3}{6} \div 3 = \frac{1}{6}$
 - A student may think or draw $\frac{1}{2}$ and cut it into 3 equal groups then determine that each part is $\frac{1}{6}$.
 - A student may think of $\frac{1}{2}$ as equivalent to $\frac{3}{6}$. Therefore, $\frac{3}{6} \div 3 = \frac{1}{6}$.

Measurement and Data (MD)

Convert like measurement units within a given measurement system.

Standard
<p>5.MD.1 – Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.</p> <p><i>Explanation:</i></p> <ul style="list-style-type: none">In 5th grade, students build on their prior knowledge of related measurement units to determine equivalent measurements. Prior to making actual conversions, they examine the units to be converted, determine if the converted amount will be more or less units than the original unit, and explain their reasoning. They use several strategies to convert measurements. When converting metric measurement, students apply their understanding of place value and decimals. In order for students to have a better understanding of the relationships between units, they need to use measuring tools in class. The number of units must relate to the size of the unit. For example, students have discovered that there are 12 inches in 1 foot and 3 feet in 1 yard. This understanding is needed to convert inches to yards. Using 12-inch rulers and yardsticks, students can see that 3 of the 12-inch rulers are equivalent to 1 yardstick ($3 \times 12 \text{ inches} = 36 \text{ inches}$; $36 \text{ inches} = 1 \text{ yard}$). Using this knowledge, students can decide whether to multiply or divide when making conversions. <p>Once students have an understanding of the relationships between units and how to do conversions, they are ready to solve multi-step problems that require conversions within the same system. Allow students to discuss methods used in solving the problems. Begin with problems that allow for renaming the units to represent the solution before using problems that require renaming to find the solution.</p> <p><i>Resources:</i></p> <ul style="list-style-type: none">Yardsticks (meter sticks) and rulers (marked with customary and metric units)Teaspoons and tablespoonsGraduated measuring cups (marked with customary and metric units)NCTM, Illuminations: Discovering Gallon Man – Students experiment with units of liquid measure used in the customary system of measurement. They practice making volume conversions in the customary system.NCTM, Illuminations: Do You Measure Up? – Students learn the basics of the metric system. They identify which units of measurement are used to measure specific objects, and they learn to convert between units within the same system.

Represent and interpret data.

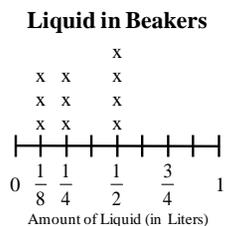
Standard

5.MD.2 – Make a line plot to display a data set of measurements in fractions of a unit ($\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Explanation: This standard provides a context for students to work with fractions by measuring objects to one-eighth of a unit. This includes length, mass, and liquid volume. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.

Examples:

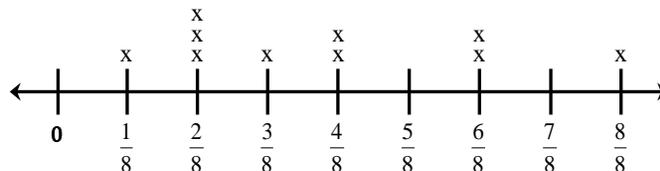
- Ten beakers, measured in liters, are filled with a liquid.



The above line plot shows the amount of liquid in liters in 10 beakers. If the liquid is redistributed equally, how much liquid would each beaker have? (This amount is the mean.)

Students apply their understanding of operations with fractions. They use either addition and/or multiplication to determine the total number of liters in the beakers. Then, the sum of the liters is shared evenly among the 10 beakers.

- Students measure objects in their desk to the nearest $\frac{1}{2}$, $\frac{1}{4}$, or $\frac{1}{8}$ of an inch and then display data collected on a line plot. How many objects measured $\frac{1}{4}$ of an inch? $\frac{1}{2}$ of an inch? If you put all the objects together end to end, what would be the total length of all the objects?



Resources:

- NCTM, Illuminations: [Fractions in Every Day Life](#) – This activity enables students to apply their knowledge about fractions to a real-life situation. It also provides a good way for teachers to assess students' working knowledge of fraction multiplication and division.

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

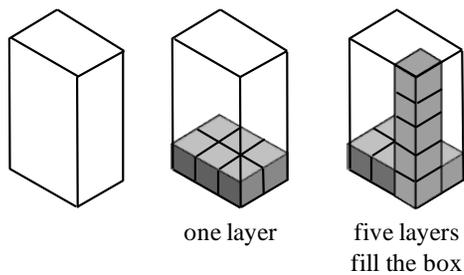
Standard

5.MD.3 – Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

- A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.
- A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.

Explanation:

- 5. MD.3, 5.MD.4, and 5. MD.5 represent the first time that students begin exploring the concept of volume. In 3rd grade, students begin working with area and covering spaces. The concept of volume should be extended from area with the idea that students are covering an area (the bottom of cube) with a layer of unit cubes and then adding layers of unit cubes on top of bottom layer (see picture below). Students should have ample experience with concrete manipulatives before moving to pictorial representations.
 - (3×2) is represented by the first layer
 - $(3 \times 2) \times 5$ represents the number of 3×2 layers
 - $(3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) = 6 + 6 + 6 + 6 + 6 = 30$, where 6 represents the size/area of one layer



Have students build a prism in layers. Then, have students determine the number of cubes in the bottom layer and share their strategies. Students should use multiplication based on their knowledge of arrays and its use in multiplying two whole numbers.

Ask what strategies can be used to determine the volume of the prism based on the number of cubes in the bottom layer. Expect responses such as “adding the same number of cubes in each layer as were on the bottom layer” or multiply the number of cubes in one layer times the number of layers.

Resources:

- Cubes, rulers (marked in standard or metric units), grid paper
- NCTM, Illuminations: [Cubes](#) – determine the volume of a box by filling it with cubes

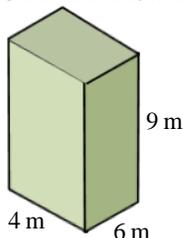
Standard

5.MD.4 – Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

Explanation: Students understand that same sized cubic units are used to measure volume. They select appropriate units to measure volume. For example, they make a distinction between which units are more appropriate for measuring the volume of a gym and the volume of a box of books. They can also improvise a cubic unit using any unit as a length (e.g., the length of their pencil). Students can apply these ideas by filling containers with cubic units (wooden cubes) to find the volume. They may also use drawings or interactive computer software to simulate the same filling process.

Example:

- What is the volume of this prism? If I know the first layer of this prism, then I can find the volume of the entire prism since I stack the layers.



5.MD.5 – Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

- Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.
- Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.
- Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Explanation: Students need multiple opportunities to measure volume by filling rectangular prisms with cubes and looking at the relationship between the total volume and the area of the base. They derive the volume formula (volume equals the area of the base times the height) and explore how this idea would apply to other prisms. Students use the associative property of multiplication and decomposition of numbers using factors to investigate rectangular prisms with a given number of cubic units.

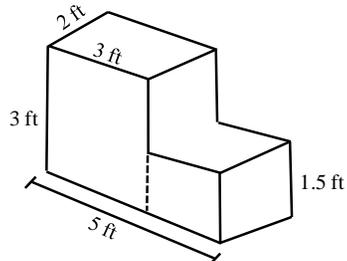
Standard

5.MD.5 Examples:

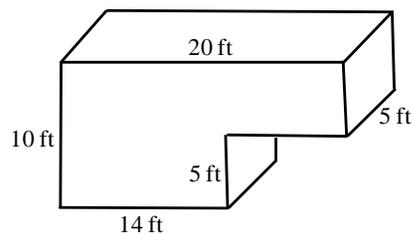
- When given 24 cubes, students make as many rectangular prisms as possible with a volume of 24 cubic units. Students build the prisms and record possible dimensions.

Length	Width	Height
1	2	12
2	2	6
4	2	3
8	3	1

- Students determine the volume of concrete needed to build the steps in the diagram below.



- A homeowner is building a swimming pool and needs to calculate the volume of water needed to fill the pool. The design of the pool is shown in the illustration below.



Geometry (G)

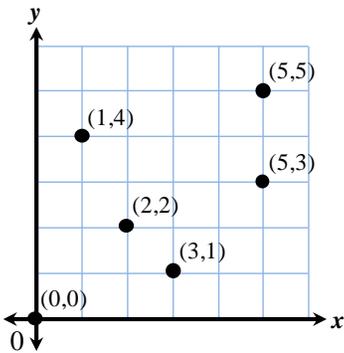
Graph points on the coordinate plane to solve real-world and mathematical problems.

Standard

5.G.1 – Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x -axis and x -coordinate, y -axis and y -coordinate).

Examples:

- Students can use a classroom size coordinate system to physically locate the coordinate point $(5, 3)$ by starting at the origin point $(0, 0)$, walking 5 units along the x -axis to find the first number in the pair (5) , and then walking up 3 units for the second number in the pair (3) . The ordered pair names a point in the plane.



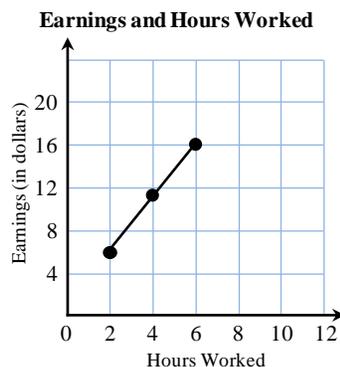
- Graph and label the points below in a coordinate system.
 - A → $(0, 0)$
 - B → $(5, 1)$
 - C → $(0, 6)$
 - D → $(2.5, 6)$
 - E → $(6, 2)$
 - F → $(4, 1)$
 - G → $(3, 0)$

Standard

5.G.2 – Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Examples:

- Sara has saved \$20. She earns \$8 for each hour she works.
 - If Sara saves all of her money, how much will she have after working 3 hours? 5 hours? 10 hours?
 - Create a graph that shows the relationship between the hours Sara worked and the amount of money she has saved.
 - What other information do you know from analyzing the graph?
- Use the graph below to determine how much money Jack makes after working exactly 9 hours.



Resources:

- Grid/graph paper
- NCTM, Illuminations: [Finding Your Way Around](#) – Students explore two-dimensional space via an activity in which they navigate the coordinate plane.
- NCTM, Illuminations: [Describe the Way](#) – Students review plotting points and labeling axes. Students generate a set of random points all located in the first quadrant.

Classify two-dimensional figures into categories based on their properties.

Standard
<p>5.G.3 – Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. <i>For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.</i></p> <p><i>Explanation:</i> Geometric properties include properties of sides (parallel, perpendicular, congruent), properties of angles (type, measurement, congruent), and properties of symmetry (point and line).</p> <p><i>Example:</i></p> <ul style="list-style-type: none">• If the opposite sides on a parallelogram are parallel and congruent, then rectangles are parallelograms.• A sample of questions that might be posed to students include:<ul style="list-style-type: none">▪ A parallelogram has 4 sides with both sets of opposite sides parallel. What types of quadrilaterals are parallelograms?▪ Regular polygons have all of their sides and angles congruent. Name or draw some regular polygons.▪ All rectangles have 4 right angles. Squares have 4 right angles so they are also rectangles. True or false?▪ A trapezoid has 2 sides parallel so it must be a parallelogram. True or false? <p><i>Resources:</i></p> <ul style="list-style-type: none">• NCTM, Illuminations: Geometric Solids – This tool allows you to learn about various geometric solids and their properties.• NCTM, Illuminations: Polygon Capture – Students classify polygons according to more than one property at a time. In the context of a game, students move from a simple description of shapes to an analysis of how properties are related.

Standard

5.G.4 – Classify two-dimensional figures in a hierarchy based on properties.

Explanation:

- Properties of figure may include:
 - Properties of sides – parallel, perpendicular, congruent, number of sides
 - Properties of angles – types of angles, congruent
- Examples:
 - A right triangle can be both scalene and isosceles, but not equilateral.
 - A scalene triangle can be right, acute, and obtuse.
- Triangles can be classified by:
 - Angles
 - ♦ Right – the triangle has one angle that measures 90° .
 - ♦ Acute: The triangle has exactly three angles that measure between 0° and 90° .
 - ♦ Obtuse: The triangle has exactly one angle that measures greater than 90° and less than 180° .
 - Sides
 - ♦ Equilateral: All sides of the triangle are the same length.
 - ♦ Isosceles: At least two sides of the triangle are the same length.
 - ♦ Scalene: No sides of the triangle are the same length.

