Delaware's

Common Core State Standards for Mathematics

Grade 3 Assessment Examples

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Delaware's Common Core State Standards for 3rd Grade Mathematics

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Grade 3 Mathematics – Unpacking the Delaware Common Core State Standards

This document is designed to help understand the Common Core State Standards (CCSS) in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

What Is the Purpose of This Document?

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

References

This document contains explanations and examples that were obtained from State Departments of Education for Kansas, Arizona, North Carolina, and Ohio with permission.

How Do I Send Feedback?

This document is helpful in understanding the CCSS but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education via rfry@doe.k12.de.us, and we will use your input to refine this document.

Operations and Algebraic Thinking (OA)

Represent and solve problems involving multiplication and division.

Standard
3.OA.1 – Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.* ***Explanation:*** • Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol "×" means "groups of" and problems such as 5×7 refer to 5 groups of 7. To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5×6) students interpret the expression using a multiplication context. They should begin to use the terms, *factor* and *product,* as they describe multiplication. ***Examples:*** • Use multiplication and division within 100 to solve word problem situations involving equal groups, arrays, and measurement. • Determine unknown factors such as: ___ $\times 6 = 42$ • Play Kids Games.com website for multiplication practice – http://www.playkidsgames.com/games/mathfact/default.htm
3.OA.2 – Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.* ***Explanation:*** • Students recognize the operation of division in two different types of situations. One situation requires determining how many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor). To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., $24 \div 6$), students interpret the expression in contexts that require both interpretations of division. Students may use interactive whiteboards to create digital models. Measurement (repeated subtraction) models focus on the question, "How many groups can you make?" A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?

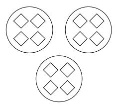
Standard
3.OA.3 – Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

Explanation:

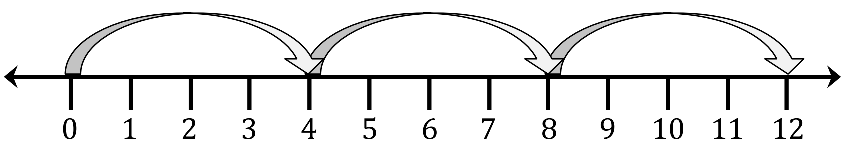
- Students use a variety of representations for creating and solving one-step word problems, e.g., numbers, words, pictures, physical objects, money, and equations. They use multiplication and division of whole numbers up to 10×10. Students explain their thinking, show their work by using at least one representation, and verify that their answer is reasonable.
- KidZone Math: Word Problems – http://www.kidzone.ws/math/wordproblems.htm#g 3wp
- Word problems may be represented in multiple ways:
  - Equations: $3 \times 4 = ?$, $4 \times 3 = ?$, $12 \div 4 = ?$, $12 \div 3 = ?$
  - Array:

    ☆☆☆☆
    ☆☆☆☆
    ☆☆☆☆

  - Equal groups:

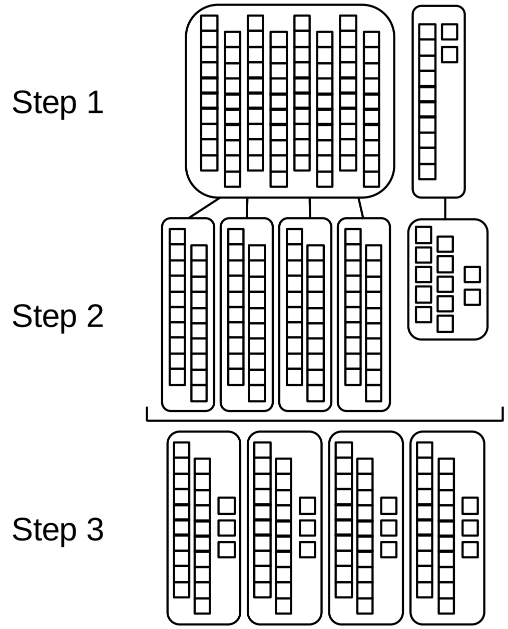


  - Repeated addition: $4 + 4 + 4$ or repeated subtraction
  - Three equal jumps forward from 0 on the number line to 12 or three equal jumps backwards from 12 to 0



Standard
3.OA.3 Examples of Division Problems

- Determining the number of objects in each share—partitive division, where the size of the groups is unknown:
  - The bag has 92 hair clips, and Laura and her three friends want to share them equally. How many hair clips will each person receive?



http://www.funbrain.com/tens/index.html

- Determining the number of shares—measurement division, where the number of groups is unknown:
  - Max the monkey loves bananas. Molly, his trainer, has 24 bananas. If she gives Max four bananas each day, how many days will the bananas last?

Starting	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6
24	$24 - 4 = 20$	$20 - 4 = 16$	$16 - 4 = 12$	$12 - 4 = 8$	$8 - 4 = 4$	$4 - 4 = 0$

Solution: The bananas will last for 6 days.

Resources: Students may use interactive whiteboards to show work and justify their thinking.

Standard
3.OA.4 – Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = __ ÷ 3, 6 × 6 = ?.*

Explanation:

- This standard is strongly connected to 3.OA.3 when students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.

  The easiest problem structure includes Unknown Product ($3 \times ? = 18$ or $18 \div 3 = 6$). The more difficult problem structures include Group Size Unknown ($3 \times ? = 18$ or $18 \div 3 = 6$) or Number of Groups Unknown ($? \times 6 = 18$, $18 \div 6 = 3$). The focus of 3.OA.4 goes beyond the traditional notion of *fact families* by having students explore the inverse relationship of multiplication and division.

  Students apply their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given $4 \times 10 = 40$, they might think:

  - 4 groups of some number is the same as 40
  - 4 times some number is the same as 40
  - I know that 4 groups of 10 is 40 so the unknown number is 10
  - The missing factor is 10 because 4 times 10 equals 40

  Equations in the form of $a \times b = c$ and $c = a \times b$ should be used interchangeably, with the unknown in different positions.

Examples:

- Solve the equations below:
  - $24 = ? \times 6$
  - $72 \div \Delta = 9$
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? Solution: $3 \times 4 = m$

Resources: Students may use interactive whiteboards to create digital models to explain and justify their thinking.

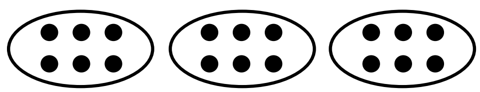
Understand properties of multiplication and the relationship between multiplication and division.

Standard
3.OA.5 – Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

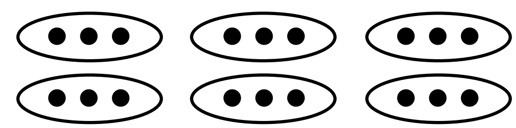
Explanation:

- Students represent expressions using various objects, pictures, words, and symbols in order to develop their understanding of properties. They multiply by 1 and 0 and divide by 1. They change the order of numbers to determine that the order of numbers does not make a difference in multiplication but does make a difference in division. Given three factors, they investigate changing the order of how they multiply the numbers to determine that changing the order does not change the product. They also decompose numbers to build fluency with multiplication.
- Models help build understanding of the commutative property. Examples:
  - $3 \times 6 = 6 \times 3$

    In the following diagram it may not be obvious that 3 groups of 6 is the same as 6 groups of 3. A student may need to count to verify this.



is the same as



  - $4 \times 3 = 3 \times 4$

    An array explicitly demonstrates the concept of the commutative property.

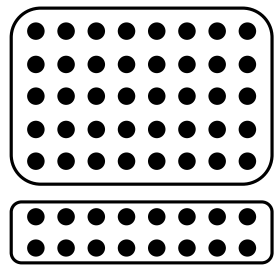
	
4 rows of 3 or 4×3	3 rows of 4 or 3×4

Standard

3.OA.5 Explanation/Examples continued

- Students are introduced to the distributive property of multiplication over addition as a strategy for using products they know to solve products they do not know. For example, if students are asked to find the product of 7×8, they might decompose 7 into 5 and 2 and then multiply 5×8 and 2×8 to arrive at $40 + 16$ or 56. Students should learn that they can decompose either of the factors. It is important to note that the students may record their thinking in different ways.

$$\begin{array}{r} 5 \times 8 = 40 \\ 2 \times 8 = \underline{16} \\ 56 \end{array}$$



$5 \times 8 = 40$

$2 \times 8 = 16$

$$\begin{array}{r} 7 \times 4 = 28 \\ 7 \times 4 = \underline{28} \\ 56 \end{array}$$

To further develop understanding of properties related to multiplication and division, students use different representations and their understanding of the relationship between multiplication and division to determine if the following types of equations are true or false.

- $7 \times 0 = 0 \times 7 = 0$ (Zero Property of Multiplication)
- $1 \times 9 = 9 \times 1 = 9$ (Multiplicative Identity Property of 1)
- $3 \times 6 = 6 \times 3 = 18$ (Commutative Property)
- $8 \div 2 = 2 \div 8$ (Students are only to determine that these are ***not*** equal)
- $2 \times 3 \times 5 = 6 \times 5$
- $10 \times 2 = 5 \times 2 \times 2$
- $2 \times 3 \times 5 = 10 \times 3$
- $0 \times 6 = 3 \times 0 \times 2$

Standard

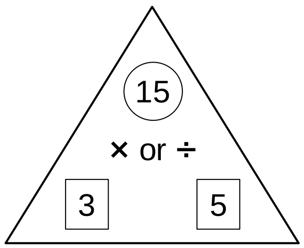
3.OA.6 – Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Explanation:

- Multiplication and division are inverse operations and that understanding can be used to find the unknown. Fact family triangles demonstrate the inverse operations of multiplication and division by showing the two factors and how those factors relate to the product and/or quotient.

Examples:

- $3 \times 5 = 15 \quad 5 \times 3 = 15$
- $15 \div 3 = 5 \quad 15 \div 5 = 3$



Explanation:

- Students use their understanding of the meaning of the equal sign as "the same as" to interpret an equation with an unknown. When given $32 \div ? = 4$, students may think:
  - 4 groups of some number is the same as 32
  - 4 times some number is the same as 32
  - I know that 4 groups of 8 is 32 so the unknown number is 8
  - The missing factor is 8 because 4 times 8 is 32
- Equations in the form of $a \div b = c$ and $c = a \div b$ need to be used interchangeably, with the unknown in different positions.

Example:

- A student knows that $2 \times 12 = 24$. How can they use that fact to determine the answer to the following question: 24 people are divided into pairs in P.E. class? How many pairs are there? Write a division equation and explain your reasoning.

Multiply and divide within 100.

Standard

3OA.7– Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
 	
Explanation:
• By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.
• “Know from memory” should not focus only on timed tests and repetitive practice, but ample experiences working with manipulatives, pictures, arrays, word problems, and numbers to internalize the basic facts (up to 9×9).
• Strategies students may use to attain fluency include:
  ▪ Multiplication by zeros and ones
  ▪ Doubles (2s facts), doubling twice (4s, e.g., 4×8 take 8 and double to 16 twice and add together)
  ▪ Tens facts (relating to place value, 5×10 is 5 tens or 50)
  ▪ Five facts (half of tens)
  ▪ Skip counting (counting groups of __ and knowing how many groups have been counted)
  ▪ Square numbers (e.g., 3×3)
  ▪ Nines (10 groups less one group, e.g., 9×3 is 10 groups of 3 minus one group of 3)
  ▪ Recalling known facts (6×7 is 6×6 plus one more group of 6)
  ▪ Turn-around facts (Commutative Property)
  ▪ Fact families (e.g., $6 \times 4 = 24; 24 \div 6 = 4; 24 \div 4 = 6; 4 \times 6 = 24$)
  ▪ Missing factors
 	
General note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

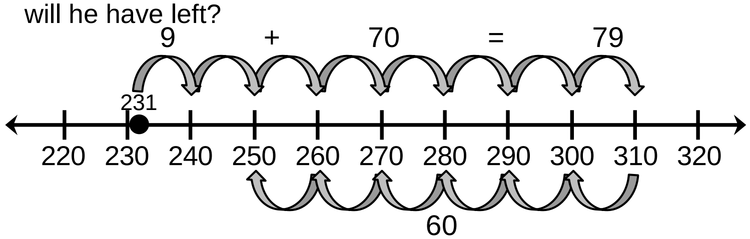
Standard
3.OA.8 – Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Explanation:

- Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects, symbol and money) and be able to choose which ones to use.

Examples:

- Jerry earned 231 points at school last week. This week he earned 79 points. If he uses 60 points to earn free time on a computer, how many points will he have left?



A student may use the number line above to describe his/her thinking, "$231 + 9 = 240$ so now I need to add 70 more. 240, 250 (10 more), 260 (20 more), 270, 280, 290, 300, 310 (70 more). Now I need to count back 60. 310, 300 (back 10), 290 (back 20), 280, 270, 260, 250 (back 60)."

A student writes the equation as $231 + 79 - 60 = m$ and uses rounding $(230 + 80 - 60)$ to estimate.

A student writes the equation $231 + 79 - 60 = m$ and calculates $79 - 60 = 19$ and then calculates $231 + 19 = m$.

- The soccer club is going on a trip to the water park. The cost of attending the trip is $63. Included in that price is $13 for lunch and the cost of 2 wristbands, one for the morning and one for the afternoon. Write an equation representing the cost of the field trip and determine the price of one wristband.

w	w	13
63		

The above diagram helps the student write the equation, $w + w + 13 = 63$. Using the diagram, a student might think, "I know that the two wristbands cost $50 ($\$63 - \$13$), so one wristband costs $25." To check for reasonableness, a student might use front-end estimation and say $60 - 10 = 50$ and $50 \div 2 = 25$.

- When students solve word problems, they use various estimation skills that include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.

Standard
3.OA.8 Examples (continued) • This standard refers to two-step word problems using the four operations. The size of the numbers should be limited to related 3rd grade standards (e.g., 3.OA.7 and 3.NBT.2). Adding and subtracting numbers should include numbers within 1,000, and multiplying and dividing numbers should include single-digit factors and products less than 100 and money. • This standard calls for students to represent problems using equations with a letter to represent unknown quantities. Example: ▪ Mike runs 2 miles a day. His goal is to run 25 miles. After 5 days, how many miles does Mike have left to run in order to meet his goal? Write an equation and find the solution ($2 \times 5 + m = 25$). • This standard refers to estimation strategies, including using compatible numbers (numbers that sum to 10, 50, or 100) or rounding. The focus in this standard is to have students use and discuss various strategies. Students should estimate during problem solving and then revisit their estimate to check for reasonableness. Estimation strategies include but are not limited to: ▪ Using benchmark numbers that are easy to compute ▪ Front-end estimation with adjusting (using the highest place value and estimating from the front end making adjustments to the estimate by taking into account the remaining amounts) ▪ Rounding and adjusting (students round down or round up and then adjust their estimate depending on how much the rounding changed the original values)
3.OA.9 – Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.* ***Explanation:*** • Students need ample opportunities to observe and identify important numerical patterns related to operations. They should build on their previous experiences with properties related to addition and subtraction. Students investigate addition and multiplication tables in search of patterns and explain why these patterns make sense mathematically. For example: ▪ Any sum of two even numbers is even. ▪ Any sum of two odd numbers is even. ▪ Any sum of an even number and an odd number is odd. ▪ The multiples of 4, 6, 8, and 10 are all even because they can all be separated into two equal groups. ▪ The doubles (2 addends the same) in an addition table fall on a diagonal while the doubles (multiples of 2) in a multiplication table fall on horizontal and vertical lines. ▪ The multiples of any number fall on a horizontal and a vertical line due to the commutative property. ▪ All the multiples of 5 end in a 0 or 5 while all the multiples of 10 end with 0. Every other multiple of 5 is a multiple of 10.

Standard

3.OA.9 Explanation (continued)

- Students also investigate a hundreds chart in search of addition and subtraction patterns. They record and organize all the different possible sums of a number and explain why the pattern makes sense.

Addend	Addend	Sum
0	20	20
1	19	20
2	18	20
3	17	20
4	16	20
•	•	•
•	•	•
•	•	•
20	0	20

Number and Operations in Base Ten (NBT)

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)

Standard

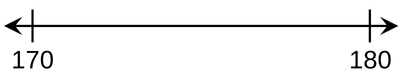
3.NBT.1 – Use place value understanding to round whole numbers to the nearest 10 or 100.

Explanation:

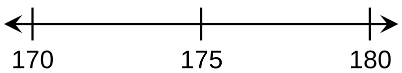
- Students learn when and why to round numbers. They identify possible answers and halfway points. Then they narrow where the given number falls between the possible answers and halfway points. They also understand that by convention if a number is exactly at the halfway point of the two possible answers, the number is rounded up.

Example:

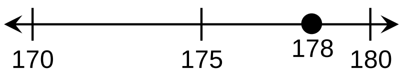
- Round 178 to the nearest 10:

Step 1: 

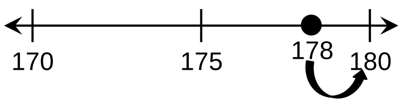
The answer is either 170 or 180.

Step 2: 

The halfway point is 175.

Step 3: 

178 is between 175 and 180.

Step 4: 

Therefore, the rounded number is 180.

Standard
3.NBT.2 – Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. ***Explanation:*** Problems should include both vertical and horizontal forms, including opportunities for students to apply the commutative and associative properties. Adding and subtracting fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Students explain their thinking and show their work by using strategies and algorithms, and explain that their answer is reasonable. An interactive whiteboard or document camera may be used to show and share student thinking. ***Example:*** • Mary read 573 pages during her summer reading challenge. She was only required to read 399 pages. How many extra pages did Mary read beyond the challenge requirements? Students may use several approaches to solve the problem including the traditional algorithm. Examples of other methods students may use are listed below: ▪ $399 + 1 = 400$, $400 + 100 = 500$, $500 + 73 = 573$, therefore $1 + 100 + 73 = 174$ pages (adding up strategy) ▪ $400 + 100 = 500$; $500 + 73 = 573$; $100 + 73 = 173$ plus 1 (for 399 to 400) is 174 (compensating strategy) ▪ $399 + 1 = 400$, 500 (that is 100 more). 510, 520, 530, 540, 550, 560, 570, (that is 70 more), 571, 572, 573 (that is 3 more), so the total is $1 + 100 + 70 + 3 = 174$ (adding by tens or hundreds strategy)
3.NBT.3 – Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations. ***Explanation:*** Students use base ten blocks, diagrams, or hundreds charts to multiply one-digit numbers by multiples of 10 from 10 to 90. They apply their understanding of multiplication and the meaning of the multiples of 10. For example, 30 is 3 tens, and 70 is 7 tens. They can interpret 2×40 as 2 groups of 4 tens or the product can also be seen as 8 groups of ten. They understand that 5×60 is 5 groups of 6 tens or 30 tens and know that 30 tens is 300. After developing this understanding, they begin to recognize the patterns in multiplying by multiples of 10. ***Resources:*** Students may use manipulatives, drawings, document camera, or interactive whiteboard to demonstrate their understanding.

Number and Operations—Fractions (NF)

Develop understanding of fractions as numbers.

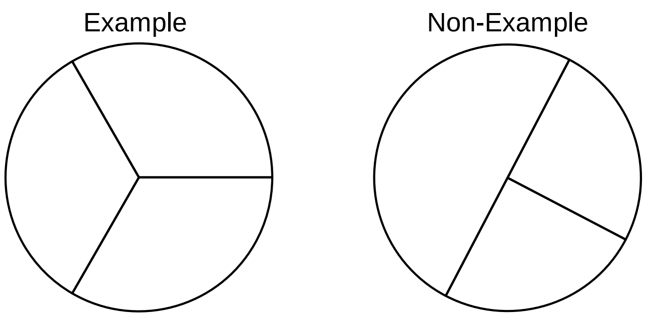
Standard
3.NF.1 – Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.

Explanation:

- Refers to the sharing of a whole being partitioned or split. Fraction models in 3rd grade include area (parts of a whole), models (circles, rectangles, squares), and number lines.

  Some important concepts related to developing understanding of fractions include:

  - Understand fractional parts must be equal-sized.

    These are thirds — These are NOT thirds

  - The number of equal parts tell how many make a whole.
  - As the number of equal pieces in the whole increases, the size of the fractional pieces decreases.
  - The size of the fractional part is relative to the whole.
    - The number of children in one-half of a **classroom** is different from the number of children in one-half of a **school** (the whole in each set is different therefore the half in each set will be different).
  - When a whole is cut into equal parts, the denominator represents the number of equal parts.
  - The numerator of a fraction is the count of the number of equal parts:
    - $\frac{3}{4}$ means that there are 3 one-fourths.
    - Students can count *one-fourth, two-fourths, three-fourths.*

Standard

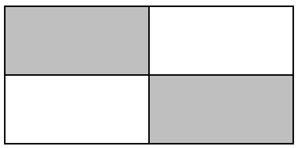
3.NF.1 Explanation (continued)

Students express fractions as fair sharing, parts of a whole, and parts of a set. They use various contexts (candy bars, fruit, and cakes) and a variety of models (circles, squares, rectangles, fraction bars, and number lines) to develop understanding of fractions and represent fractions. Students need many opportunities to solve word problems that require fair sharing.

- To develop understanding of fair shares, students first participate in situations where the number of objects is greater than the number of children and then progress into situations where the number of objects is less than the number of children.

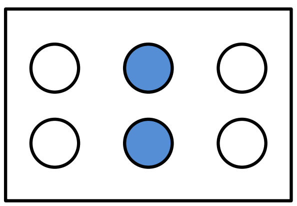
Examples:

- Four children share six brownies so that each child receives a fair share. How many brownies will each child receive?
- Six children share four brownies so that each child receives a fair share. What portion of each brownie will each child receive?
- What fraction of the rectangle is shaded? How might you draw the rectangle in another way but with the same fraction shaded?

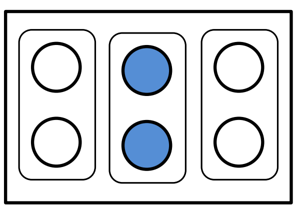


Solution: $\frac{2}{4}$ or $\frac{1}{2}$

- What fraction of each set is blue?



Solution: $\frac{2}{6}$



Solution: $\frac{1}{3}$

Standard

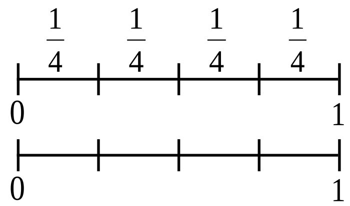
3.NF.2 – Understand a fraction as a number on the number line; represent fractions on a number line diagram.

a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.

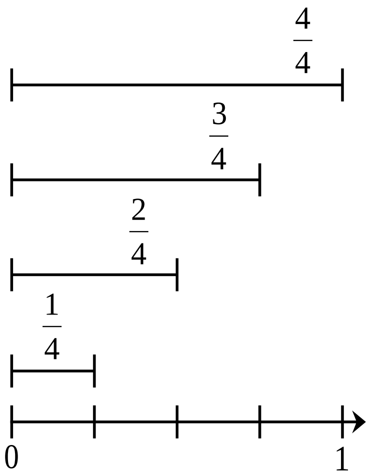
b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off *a* lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.

Explanation: Students transfer their understanding of parts of a whole to partition a number line into equal parts. There are two new concepts addressed in this standard which students should have time to develop.

- On a number line from 0 to 1, students can partition (divide) it into equal parts and recognize that each segmented part represents the same length.



- Students label each fractional part based on how far it is from zero to the endpoint.



Resources: An interactive whiteboard may be used to help students develop these concepts.

Standard
3.NF.3 – Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 =* $\frac{3}{1}$*; recognize that* $\frac{6}{1}$ *= 6; locate* $\frac{4}{4}$ *and 1 at the same point of a number line diagram.* d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. ***Explanation:*** • An important concept when comparing fractions is to look at the size of the parts and the number of the parts. For example, $\frac{1}{8}$ is smaller than $\frac{1}{2}$ because when 1 whole is cut into 8 pieces, the pieces are much smaller than when 1 whole is cut into 2 pieces. • Students recognize when examining fractions with common denominators, the wholes have been divided into the same number of equal parts. So the fraction with the larger numerator has the larger number of equal parts. $\frac{2}{6} < \frac{5}{6}$ • To compare fractions that have the same numerator but different denominators, students understand that each fraction has the same number of equal parts but the size of the parts are different. They can infer that the same number of smaller pieces is less than the same number of bigger pieces. $\frac{3}{8} < \frac{3}{4}$

Measurement and Data (MD)

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

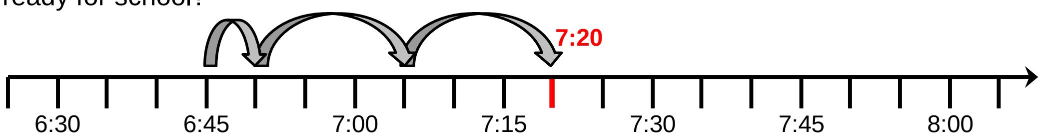
Standard

3.MD.1 – Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

Explanation: Students in 2nd grade learned to tell time to the nearest five minutes. In 3rd grade, they extend telling time to the minute and measure elapsed time both in and out of context using clocks and number lines. Students also solve for elapsed time including word problems. Students could use pre-determined number lines (intervals every 5 or 15 minutes) or open number lines (intervals determined by students).

Example:

- Tonya wakes up at 6:45 a.m. It takes her 5 minutes to shower, 15 minutes to get dressed, and 15 minutes to eat breakfast. What time will she be ready for school?



Resources: Students may use an interactive whiteboard to demonstrate understanding and justify their thinking.

3.MD.2 – Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm^3 and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Table 2).

Explanation:

- Students need multiple opportunities weighing classroom objects and filling containers to help them develop a basic understanding of the size and weight of a liter, a gram, and a kilogram. Milliliters may also be used to show amounts that are less than a liter.
- Foundational understandings to help with measure concepts:
  - Understand that larger units can be subdivided into equivalent units (partition).
  - Understand that the same unit can be repeated to determine the measure (iteration).
  - Understand the relationship between the size of a unit and the number of units needed (compensatory principal).

Example:

- Students identify 5 things that weigh about one gram. They record their findings with words and pictures. (Students can repeat this for 5 grams and 10 grams.) This activity helps develop gram benchmarks. One large paperclip weighs about one gram. A box of large paperclips (100 clips) weighs about 100 grams, so 10 boxes would weigh one kilogram.

Represent and interpret data.

Standard
3.MD.3 – Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

Explanation: Students should have opportunities reading and solving problems using scaled graphs *before* being asked to draw one. The following graphs all use five as the scale interval, but students should experience different intervals to further develop their understanding of scale graphs and number facts.

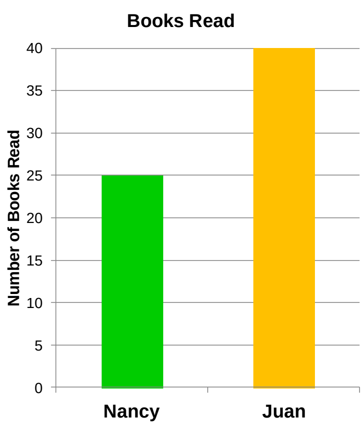
Examples:

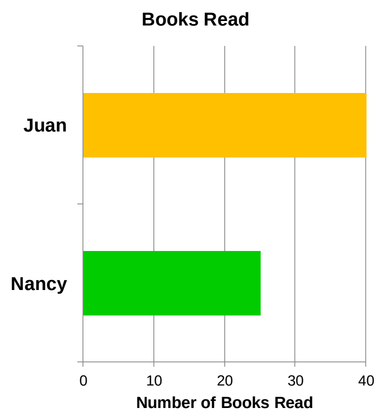
- Pictographs: Scaled pictographs include symbols that represent multiple units. Below is an example of a pictograph with symbols that represent multiple units. Graphs should include a title, categories, category label, key, and data.

Number of Books Read	
Nancy	✧ ✧ ✧ ✧ ✧
Juan	✧ ✧ ✧ ✧ ✧ ✧ ✧ ✧
✧ = 5 Books	

How many more books did Juan read than Nancy did?

- Single Bar Graphs: Students use both horizontal and vertical bar graphs. Bar graphs include a title, scale, scale label, categories, category label, and data.

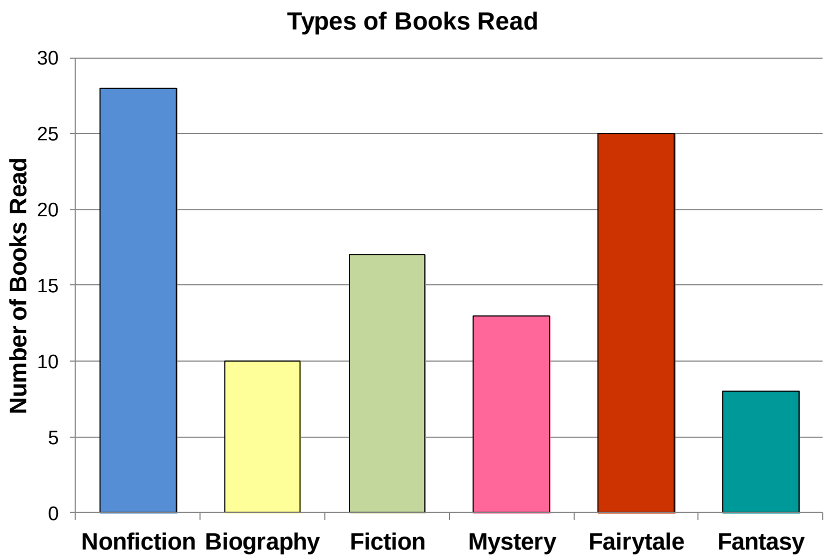




Standard

3.MD.3 Examples (continued)

- Analyze and interpret data:



- How many more nonfiction books were read than fantasy books?
- Did more people read biography and mystery books or fiction and fantasy books?
- About how many books in all genres were read?
- Using the data from the graphs, what type of book was read more often than a mystery but less often than a fairytale?
- What interval was used for this scale?
- What can we say about types of books read? What is a typical type of book read?
- If you were to purchase a book for the class library, which would be the best genre? Why?

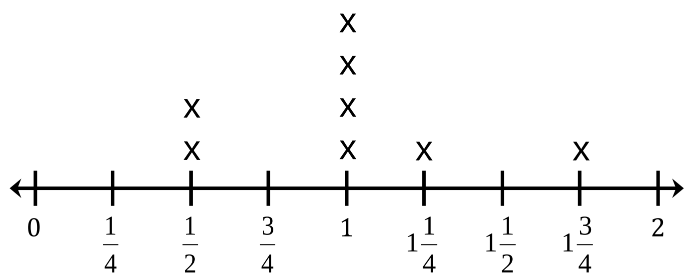
Standard

3.MD.4 – Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Explanation:

- Students in 2nd grade measured length in whole units using both metric and U.S. customary systems. It is important to review with students how to read and use a standard ruler including details about halves and quarter marks on the ruler. Students should connect their understanding of fractions to measuring to one-half and one-quarter inch.
- Third graders need many opportunities measuring the length of various objects in their environment. Some important ideas related to measuring with a ruler are:
  - The starting point of where one places a ruler to begin measuring.
  - Measuring is approximate. Items that students measure will not always measure exactly $\frac{1}{4}$, $\frac{1}{2}$, or 1 inch. Students will need to decide on an appropriate estimate length.
  - Making paper rulers and folding to find the half and quarter marks will help students develop a stronger understanding of measuring length.
- Students generate data by measuring and create a line plot to display their findings. An example of a line plot is shown below:

Number of Objects Measured



Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

Standard

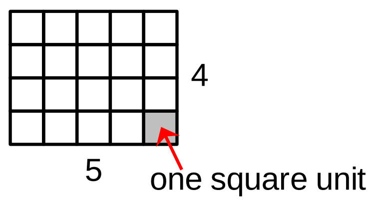
3.MD.5 – Recognize area as an attribute of plane figures and understand concepts of area measurement.

a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.

Explanation:

- Students develop understanding of using square units to measure area by:
  - Using different sized square units.
  - Filling in an area with the same sized square units and counting the number of square units.
  - An interactive whiteboard would allow students to see that square units can be used to cover a plane figure.



3.MD.6 – Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

Resources: Using different sized paper, students can explore the areas measured in square centimeters, square inches, square feet, yards, or meters. An interactive whiteboard may also be used to display and count the unit squares (area) of a figure.

3.MD.7 – Relate area to the operations of multiplication and addition.

a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

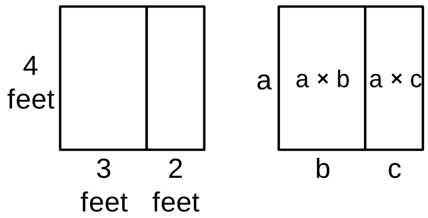
Explanation: Students tile areas of rectangles, determine the area, record the length and width of the rectangle, investigate the patterns in the numbers, and discover that the area is the length times the width.

Standard

3.MD.7 Examples:

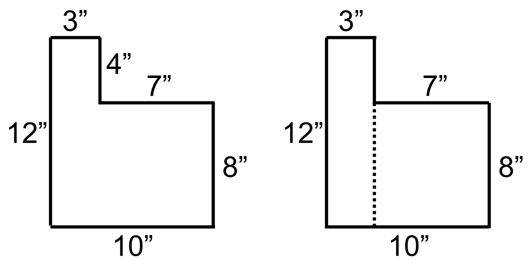
- Joe and John made a poster that was 4 feet by 3 feet. Mary and Amir made a poster that was 4 feet by 2 feet. They placed their posters on the wall side-by-side so that that there was no space between them. How much area will the two posters cover?

  Students use pictures, words, and numbers to explain their understanding of the distributive property in this context.



$$4 \times 3 + 4 \times 2 = 20$$
$$4(3 + 2) = 20$$
$$4 \times 5 = 20$$

- Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



Area is $12 \times 3 + 8 \times 7 = 92\ sq\ inches$

- To find the area, one could count the squares or multiply $3 \times 4 = 12$.

1	2	3	4
5	6	7	8
9	10	11	12

After counting units, then count length and width to multiply together to get area.

Standard

3.MD.8 – Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Explanation:

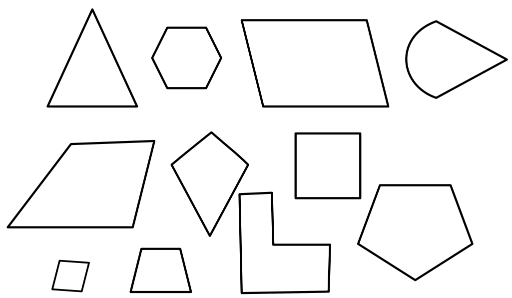
- Students develop an understanding of the concept of perimeter by walking around the perimeter of a room, using rubber bands to represent the perimeter of a plane figure on a geoboard, or tracing around a shape on an interactive whiteboard. They find the perimeter of objects, use addition to find perimeters, and recognize the patterns that exist when finding the sum of the lengths and widths of rectangles.
- Students use geoboards, tiles, and graph paper to find all the possible rectangles that have a given perimeter (e.g., find the rectangles with a perimeter of 14 cm). They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles.
- Given a perimeter and a length or width, students use objects or pictures to find the missing length or width. They justify and communicate their solutions using words, diagrams, pictures, numbers, and an interactive whiteboard.
- Students use geoboards, tiles, graph paper, or technology to find all the possible rectangles with a given area (e.g., find the rectangles that have an area of 12 square units). They record all the possibilities using dot or graph paper, compile the possibilities into an organized list or a table, and determine whether they have all the possible rectangles. Students then investigate the perimeter of the rectangles with an area of 12.

Area	Length	Width	Perimeter
12 sq. in.	1 in.	12 in.	26 in.
12 sq. in.	2 in.	6 in.	16 in.
12 sq. in.	3 in.	4 in.	14 in.
12 sq. in.	4 in.	3 in.	14 in.
12 sq. in.	6 in.	2 in.	16 in.
12 sq. in.	12 in.	1 in.	26 in.

The patterns in the chart allow the students to identify the factors of 12, connect the results to the commutative property, and discuss the differences in perimeter within the same area. This chart can also be used to investigate rectangles with the same perimeter. It is important to include squares in the investigation.

Geometry (G)

Reason with shapes and their attributes.

Standard
3.G.1 – Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories. ***Explanation:*** In 2^{nd} grade, students identify and draw triangles, quadrilaterals, pentagons, and hexagons. In 3^{rd} grade, students build on this experience and further investigate quadrilaterals (technology may be used during this exploration). Students recognize shapes that are and are not quadrilaterals by examining the properties of the geometric figures. They conceptualize that a quadrilateral must be a closed figure with four straight sides and begin to notice characteristics of the angles and the relationship between opposite sides. Students should be encouraged to provide details and use proper geometric vocabulary when describing the properties of quadrilaterals. They sort geometric figures (see examples below) and identify squares, rectangles, and rhombi as quadrilaterals. 

Standard

3.G.2 – Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Explanation: Given a shape, students partition it into equal parts, recognizing that these parts all have the same area. They identify the fractional name of each part and are able to partition a shape into parts with equal areas in several different ways.

$\frac{1}{4}$	$\frac{1}{4}$
$\frac{1}{4}$	$\frac{1}{4}$

$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$
$\frac{1}{4}$

$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Appendix – *Common Multiplication and Division Situations*[1]

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown (How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ and $18 \div 3 = ?$	$? \times 6 = 18$ and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example:* You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example:* You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example:* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays[2], Areas[3]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example:* What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example:* A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example:* A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example:* A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example:* A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	How many times as much does the red hat cost as the blue hat? *Measurement example:* A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$ and $p \div a = ?$	$? \times b = p$ and $p \div b = ?$

[1] The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
[2] The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.
[3] Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.