Delaware's
Common Core State Standards for Mathematics
Grade 6 Assessment Examples

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July 2012

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Grade 6 Mathematics – Unpacking the Delaware Common Core State Standards

This document is designed to help understand the Common Core State Standards (CCSS) in providing examples that show a range of format and complexity. It is a work in progress, and it does not represent all aspects of the standards.

What Is the Purpose of This Document?

This document may be used to facilitate discussion among teachers and curriculum staff and to encourage coherence in the expectations. This document, along with ongoing professional development, is one of many resources used to understand and teach the Delaware Common Core State Standards.

This document contains descriptions of what each standard means and what a student is expected to know, understand, and be able to do. This is meant to eliminate misinterpretation of the standards.

References

This document contains explanations and examples that were obtained from State Departments of Education for Arizona, North Carolina, and Ohio with their permission.

How Do I Send Feedback?

This document is helpful in understanding the CCSS but is an evolving document where more comments and examples might be necessary. Please feel free to send feedback to the Delaware Department of Education via rfry@doe.k12.de.us, and we will use your input to refine this document.

## *Ratios and Proportional Relationships (RP)*

Understand ratio concepts and use ratio reasoning to solve problems.

Standard
6.RP.1 – Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*

Explanations:

- Proportional reasoning is a process that requires instruction and practice. It does not develop over time on its own. Grade 6 is the first of several years in which students develop this multiplicative thinking.

- Students develop the understanding that ratio is a comparison of two numbers or quantities. Ratios that are written as part-to-whole are comparing a specific part to the whole. Fractions and percents are examples of part-to-whole ratios. Fractions are written as the part being identified compared to the whole amount. A percent is the part identified compared to the whole (100). Provide students with multiple examples of ratios, fractions, and percents of this type. For example, the number of girls in the class (12) to the number of students in the class (28) is the ratio 12 to 28.

- A ratio is a comparison of two quantities which can be written as: a to b, $\frac{a}{b}$, or $a:b$.

- A rate is a ratio where two measurements are related to each other. When discussing measurement of different units, the word rate is used rather than ratio. Understanding rate, however, is complicated and there is no universally accepted definition. When using the term rate, contextual understanding is critical. Students need many opportunities to use models to demonstrate the relationships between quantities before they are expected to work with rates numerically.

- A comparison of 8 black circles to 4 white circles can be written as the ratio of 8:4 and can be regrouped into 4 black circles to 2 white circles (4:2), and 2 black circles to 1 white circle (2:1).
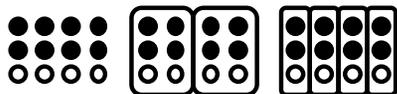
Students should be able to identify all these ratios and describe them using, "For every…, there are…."

Instructional Resources/Tools:

- 100 grids (10 x 10) for modeling percents

- Ratio tables – to use for proportional reasoning

- Bar models – for example, 4 red bars to 6 blue bars as a visual representation of a ratio and then

- Expand the number of bars to show other equivalent ratios

Standard

- *Something Fishy* – Students estimate the size of a large population by applying the concepts of ratio and proportion through the capture-recapture statistical procedure
- *How Many Noses Are in Your Arm?* – Students apply the concept of ratio and proportion to determine the length of the Statue of Liberty's torch-bearing arm.

6.RP.2 – Understand the concept of a unit rate a/b associated with a ratio *a:b* with *b* ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."* (Expectations for unit rates in this grade are limited to non-complex fractions.)

Explanations:

- A unit rate compares a quantity in terms of one unit of another quantity. Students will often use unit rates to solve missing value problems. Cost per item or distance per time unit are common unit rates; however, students should be able to flexibly use unit rates to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates are reciprocals as in the first example. It is not intended that this be taught as an algorithm or rule, because at this level, students should primarily use reasoning to find these unit rates.
- In Grade 6, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

Examples:

- On a bicycle you can travel 20 miles in 4 hours. What are the unit rates in this situation (the distance you can travel in 1 hour and the amount of time required to travel 1 mile)?

    - Solution: You can travel 5 miles in 1 hour written as $\frac{5\ mi}{1\ hr}$, and it takes $\frac{1}{5}$ of an hour to travel each mile, written as $\frac{\frac{1}{5}\ hr}{1\ mi}$. Students can represent the relationship between 20 miles and 4 hours.

    1 mile

    1 hour
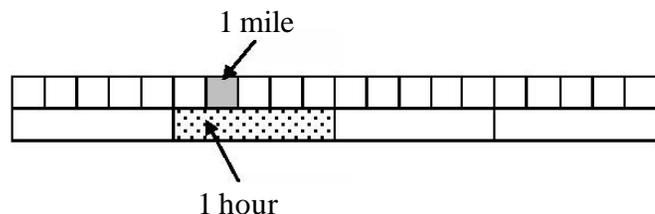
- A simple modeling clay recipe calls for 1 cup of cornstarch, 2 cups of salt, and 2 cups of boiling water. How many cups of cornstarch are needed to mix with each cup of salt?

Standard

6.RP.3 – Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Examples:

• At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54? To find the price of 1 book, divide $18 by 3. One book is $6. To find the price of 7 books, multiply $6, the cost of one book, times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates, and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally and vertically. Red numbers indicate solutions.

| Number of Books	Cost
1	6
3	18
7	42
9	54

• Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain how you determined your answer.

Number of Books	Cost
4	20
8	40

Standard

6.RP.3 Examples (continued)

- To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$.

- The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane. Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:
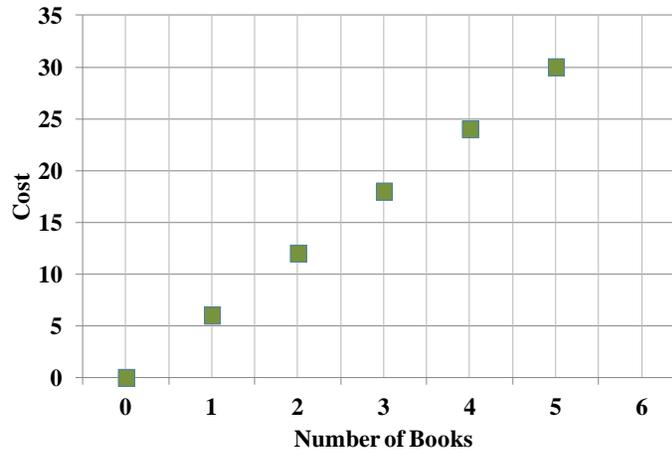
- Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

There are several strategies that students could use to determine the solution to this problem.

- Add quantities from the table to 24 feet (9 feet and 15 feet); there, the number of yards must be 8 yeards (3 yards and 5 yards).
- Use multiplication to find 24 feet:
  - $3 \, feet \times 8 = 24 \, feet$; therefore, $1 \, yard \times 8 = 8 \, yards$ or
  - $6 \, feet \times 4 = 24 \, feet$; therefore, $2 \, yards \times 4 = 8 \, yards$.

- Compare the number of black to white circles. If the ratio remains the same, how many black circles will you have if you have 60 white circles?
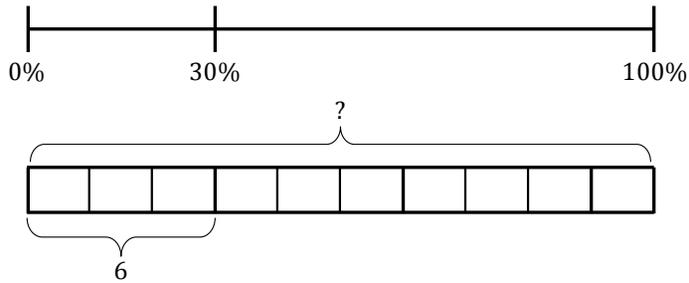
● ● ● ● ○ ○ ○

Black	4	40	20	60	?
White	3	30	15	45	60

Standard

6.RP.3 Examples (continued)

- If 6 is 30% of a value, what is that value? (Solution: 20)

Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent). For example, to find 40% of 30, students could use a 10×10 grid to represent the whole (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40×0.3, which equals 12.

Students also find the whole, given a part and the percent. For example, if 25% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream? Students can reason that if 25% is 6 and 100% is 4 times the 25%, then 6 times 4 would give 24 students in Mrs. Rutherford's class.

A credit card company charges 17% interest on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If your bill totals $450 for this month, how much interest would you have to pay if you let the balance carry to the next month? Show the relationship on a graph and use the graph to predict the interest charges for a $300 balance.

Charges	$1.00	$50.00	$100.00	$200.00	$450.00
Interest	$0.17	$8.50	$17.00	$34.00	?

Part-to-part ratios are used to compare two parts. For example, the number of girls in the class (12) compared to the number of boys in the class (16) is the ratio the ratio 12 to 16. This form of ratio is often used to compare the event that can happen to the event that cannot happen.

Rates, a relationship between two units of measure, can be written as ratios, such as miles per hour, ounces per gallon, and students per bus. For example, 3 cans of pudding cost $2.48 at Store A and 6 cans of the same pudding costs $4.50 at Store B. Which store has the better buy on these cans of pudding? Various strategies could be used to solve this problem:

- A student can determine the unit cost of 1 can of pudding at each store and compare.
- A student can determine the cost of 6 cans of pudding at Store A by doubling $2.48.
- A student can determine the cost of 3 cans of pudding at Store B by taking $\frac{1}{2}$ of $4.50.

Standard

6.RP.3 Examples (continued)

Using ratio tables develops the concept of proportion. By comparing equivalent ratios, the concept of proportional thinking is developed and many problems can be easily solved.

Store A	
3 cans	6 cans
$2.48	$4.96

Store B	
6 cans	3 cans
$4.50	$2.25

Students should also solve real-life problems involving measurement units that need to be converted. Representing these measurement conversions with models such as ratio tables, t-charts, or double number-line diagrams will help students internalize the size relationships between same system measurements and relate the process of converting to the solution of a ratio.

- Multiplicative reasoning is used when finding the missing element in a proportion. For example, use 2 cups of syrup to 5 cups of water to make fruit punch. If 6 cups of syrup are used to make punch, how many cups of water are needed?

$$\frac{2}{5} = \frac{6}{x}$$

Recognize that the relationship between 2 and 6 is 3 times: $2 \bullet 3 = 6$. To find x, the relationship between 5 and x must also be 3 times. To find x, the relationship between 5 and x must also be 3 times: $3 \bullet 5 = x$; therefore, $x = 15$

The final proportion is: $\frac{2}{5} = \frac{6}{15}$

- Other ways to illustrate ratios that will help students see the relationships follow. Begin written representation of ratios with the words "out of" or "to" before using the symbolic notation of the colon and then the fraction bar. For example, 3 out of 7, 3 to 5, 6:7, and then $\frac{4}{5}$. Use skip counting as a technique to determine if ratios are equal.

- Labeling units helps students organize the quantities when writing proportions.

$$\frac{3\ eggs}{2\ cups\ of\ flour} = \frac{w\ eggs}{8\ cups\ of\ flour}$$

- Using hue/color intensity is a visual way to examine ratios of part-to-part. Students can compare the intensity of the color green and relate that to the ratio of colors used. For example, have students mix green paint into white paint in the following ratios: 1 part green to 5 parts white, 2 parts green to 3 parts white, and 3 parts green to 7 parts white. Compare the green color intensity with their ratios.

The Number System (NS)

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

Standard
6.NS.1 – Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Resources: Contexts and visual models can help students to understand quotients of fractions and begin to develop the relationship between multiplication and division. Model development can be facilitated by building from familiar scenarios with whole or friendly number dividends or divisors. Computing quotients of fractions build upon and extends student understandings developed in Grade 5. Students make drawings, model situations with manipulatives, or manipulate computer-generated models.
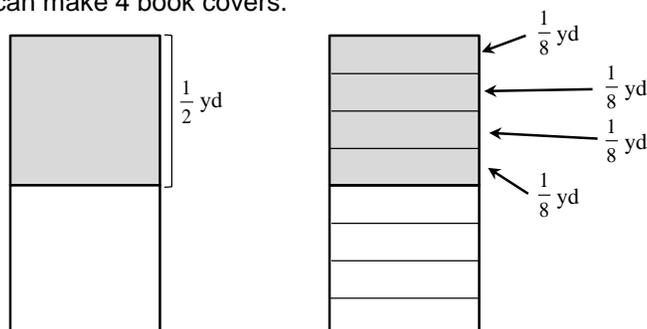
Examples:

- Three people share $\frac{1}{2}$ pound of chocolate. How much of a pound of chocolate does each person get?

  - Solution: Each person gets $\frac{1}{6}$ pound of chocolate.

  - Manny has $\frac{1}{2}$ yard of fabric to make book covers. Each book cover is made from $\frac{1}{8}$ yard of fabric. How many book covers can Manny make?
   Solution: Manny can make 4 book covers.

Standard

- Represent $\frac{1}{2} \div \frac{2}{3}$ in a problem context and draw a model to show your solution.

  Context: You are making a recipe that calls for $\frac{2}{3}$ cup of yogurt. You have $\frac{1}{2}$ cup of yogurt from a snack pack. How much of the recipe can you make?

  Explanation of Model:

  The first model below shows $\frac{1}{2}$ cup. The shaded squares in all three models show $\frac{1}{2}$ cup. The second model shows $\frac{1}{2}$ cup and also shows $\frac{1}{3}$ cups horizontally. The third model shows $\frac{1}{2}$ cup moved to fit in only the area shown by $\frac{2}{3}$ of the model. $\frac{2}{3}$ is the new referent unit (whole). 3 out of 4 squares in the $\frac{2}{3}$ portion are shaded. A $\frac{1}{2}$ cup is only $\frac{3}{4}$ of a $\frac{2}{3}$ cup portion, so you can only make $\frac{3}{4}$ of the recipe.
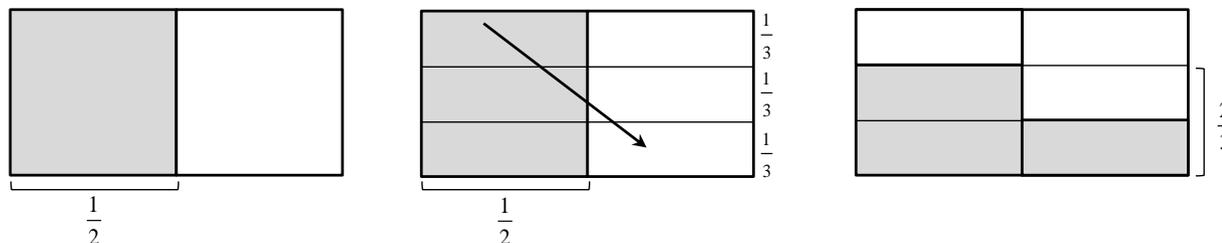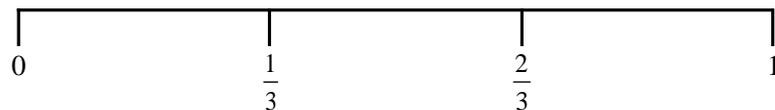

Students also write contextual problems for fraction division problems. For example, the problem $\frac{2}{3} \div \frac{1}{6}$ can be illustrated with the following word problem:

Susan has $\frac{2}{3}$ of an hour left to make cards. It takes her about $\frac{1}{6}$ of an hour to make each card. About how many can she make?
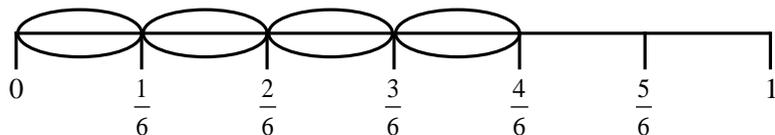
This problem can be modeled using a number line.

1. Start with a number line divided into thirds.

Standard

2. The problem wants to know how many sixths are in two-thirds. Divide each third in half to create sixths.

$$0 \qquad \frac{1}{6} \qquad \frac{2}{6} \qquad \frac{3}{6} \qquad \frac{4}{6} \qquad \frac{5}{6} \qquad 1$$

3. Each circled part represents $\frac{1}{6}$. There are 4 sixths in two-thirds; therefore, Susan can make 4 cards.

For example, $12 \div 3$ means: how many groups of three would make 12? Or, how many in each of 3 groups would make 12? Thus, $\frac{7}{2} \div \frac{1}{4}$ can be solved the same way. How many groups of $\frac{1}{4}$ make $\frac{7}{2}$? Or, how many objects in a group when $\frac{7}{2}$ fills $\frac{1}{4}$? Creating the picture that represents this problem makes seeing and proving the solutions easier.

- Set the problem in context and represent the problem with a concrete or pictorial model: $\frac{5}{4} \div \frac{1}{2}$.
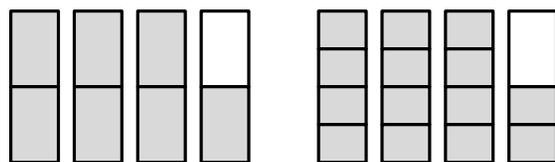
$\frac{5}{4}$ cups of nuts fills $\frac{1}{2}$ of a container. How many cups of nuts will fill the entire container?

Teaching "invert and multiply" without developing an understanding of why it works first leads to confusion as to when to apply the shortcut. Learning how to compute fraction division problems is one part, being able to relate the problems to real-world situations is important. Providing opportunities to create stories for fraction problems or writing equations for situations is needed.

Resources:

- Models for Multiplying and Dividing Fractions – Annenberg Learner Foundation teacher resource that shows how the area model can be used in multiplication and division of fractions. A section on the relationship to decimals is included.

- Fractions – Rectangle Multiplication – National Library of Virtual Manipulatives virtual manipulative to graphically demonstrate, explore, and practice multiplying fractions.

Compute fluently with multi-digit numbers and find common factors and multiples.

Standard
6.NS.2 – Fluently divide multi-digit numbers using the standard algorithm.

Explanations:

- Procedural fluency is defined by the Common Core as "skill in carrying out procedures flexibly, accurately, efficiently and appropriately." In the elementary grades, students were introduced to division through concrete models and various strategies to develop an understanding of this mathematical operation (limited to 4-digit numbers divided by 2-digit numbers). In 6th grade, students become fluent in the use of the standard division algorithm. This understanding is foundational for work with fractions and decimals in 7th grade.

- Students are expected to fluently and accurately divide multi-digit whole numbers. Divisors can be any number of digits at this grade level. As students divide, they should continue to use their understanding of place value to describe what they are doing. When using the standard algorithm, students' language should reference place value. For example, when dividing 32 into 8456, as they write a 2 in the quotient they should say, "there are 200, 32s in 8456" and could write 6400 beneath the 8456 rather than only writing 64.

$\begin{array}{r} 2 \\ 32\overline{)8456} \end{array}$	▪ There are 200, 32s in 8456.
$\begin{array}{r} 2 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	▪ 200 times 32 is 6400. ▪ 8456 minus 6400 is 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \end{array}$	▪ There are 60, 32s in 2056.
$\begin{array}{r} 26 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \end{array}$	▪ 60 times 32 is 1920. ▪ 2056 minus 1920 is 136.

Standard

6.NS.2 Explanation (continued)

$$\begin{array}{r} 264 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \\ -128 \end{array}$$	▪ There are 4, 32s in 136. ▪ 4 times 32 is 128.
$$\begin{array}{r} 264 \\ 32\overline{)8456} \\ -6400 \\ \hline 2056 \\ -1920 \\ \hline 136 \\ -128 \\ \hline 8 \end{array}$$	▪ The remainder is 8. There is not a full 32 in 8; only part of a 32 in 8. ▪ This can also be written as $\frac{8}{32}$ or $\frac{1}{4}$. This is $\frac{1}{4}$ of a 32 in 8. ▪ $8456 = 264 \times 32 + 8$

6.NS.3 – Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

Explanation: The use of estimation strategies supports student understanding of operating on decimals.

Example:

• First, students estimate the sum and then find the exact sum of 14.4 and 8.75. An estimate of the sum might be $14 + 9$ or 23. Students may also state if their estimate is low or high. They would expect their answer to be greater than 23. They can use their estimates to self-correct.

Answers of 10.19 or 101.9 indicate that students are not considering the concept of place value when adding (adding tenths to tenths or hundredths to hundredths), whereas answers like 22.125 or 22.79 indicate that students are having difficulty understanding how the four-tenths and seventy-five hundredths fit together to make one whole and 25 hundredths.

Students use the understanding they developed in 5th grade related to the patterns involved when multiplying and dividing by powers of 10 to develop fluency with operations with multi-digit decimals.

Standard

6.NS.4 – Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4(9+2).*

Examples:

- What is the greatest common factor (GCF) of 24 and 36? How can you use factor lists or the prime factorizations to find the GCF?
  - Solution: $2^2 \times 3 = 12$ – Students should be able to explain that both 24 and 36 have 2 factors of 2 and one factor of 3, thus $2 \times 2 \times 2 = 3$ is the greatest common factor.
- What is the least common multiple (LCM) of 12 and 8? How can you use multiple lists or the prime factorizations to find the LCM?
  - Solution: $2^3 \times 3 = 24$ – Students should be able to explain that the least common multiple is the smallest number that is a multiple of 12 and a multiple of 8. To be a multiple of 12, a number must have 2 factors of 2 and one factor of 3 ($2 \times 2 \times 3$). To be a multiple of 8, a number must have 3 factors of 2 ($2 \times 2 \times 2$). Thus, the least common multiple of 12 and 8 must have 3 factors of 2 and one factor of 3 ($2 \times 2 \times 2 \times 3$).
- Rewrite $84 + 28$ by using the distributive property. Have you divided by the largest common factor? How do you know?
- Given various pairs of addends using whole numbers from 1–100, students should be able to identify if the two numbers have a common factor. If they do, they identify the common factor and use the distributive property to rewrite the expression. They prove that they are correct by simplifying both expressions.
  - $27 + 36 = 9(3 + 4)$
    $63 = 9 \times 7$
    $63 = 63$
  - $31 + 80$ (There are no common factors. I know that because 31 is a prime number—it only has 2 factors, 1 and 31. I know that 31 is not a factor of 80 because $2 \times 31 = 62$ and $3 \times 31 = 93$.)

Apply and extend previous understandings of numbers to the system of rational numbers.

Standard

6.NS.5 – Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
 	
Explanation: Students use rational numbers (fractions, decimals, and integers) to represent real-world contexts and understand the meaning of 0 in each situation. For example, 25 feet below sea level can be represented as –25; 25 feet above sea level can be represented as +25. In this scenario, 0 would represent sea level.
 	
Examples:
 	
• On the same winter morning, the temperature is $-28°F$ in Anchorage, Alaska and $65°F$ in Miami, Florida. How many degrees warmer was it in Miami than in Anchorage on that morning?
 	
• Denver, Colorado is called "The Mile High City" because its elevation is 5280 feet above sea level. Someone tells you that the elevation of Death Valley, California is –282 feet.
   a. Is Death Valley located above or below sea level? Explain.
   b. How many feet higher is Denver than Death Valley?
   c. What would your elevation be if you were standing near the ocean?

6.NS.6 – Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane. *Explanation:* Number lines can be used to show numbers and their opposites. Both 3 and –3 are 3 units from zero on the number line. Graphing points and reflecting across zero on a number line extends to graphing and reflecting points across axes on a coordinate grid. The use of both horizontal and vertical number line models facilitates the movement from number lines to coordinate grids. *Example:* • Graph the following points in the correct quadrant of the coordinate plane. If you reflected each point across the x-axis, what are the coordinates of the reflected points? What similarities do you notice between coordinates of the original point and the reflected point? $\left(\frac{1}{2}, -3\frac{1}{2} \right)$ $\left(-\frac{1}{2}, -3 \right)$ $(0.25, -0.72)$

Standard
6.NS.7 – Understand ordering and absolute value of rational numbers.

a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret −3 > −7 as a statement that −3 is located to the right of −7 on a number line oriented from left to right.*

b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write −3℃ > −7℃ to express the fact that −3℃ is warmer than −7℃.*

c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of −30 dollars, write |−30| = 30 to describe the size of the debt in dollars.*

d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than −30 dollars represents a debt greater than 30 dollars.*
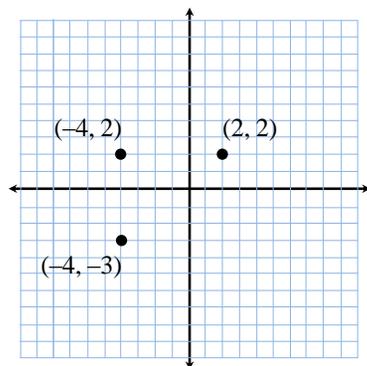
Explanations:

- Students identify the absolute value of a number as the distance from 0 but understand that although the value of −7 is less than −3, the absolute value (distance) of −7 is greater than the absolute value (distance) of −3. Students use inequalities to express the relationship between two rational numbers, understanding that the value of numbers is smaller moving to the left on a number line. For example, $-4\frac{1}{2} < -2$ because $-4\frac{1}{2}$ is located to the left of −2 on the number line.

- Students write statements using < or > to compare rational numbers in context. However, explanations should reference the context rather than "less than" or "greater than." For example, the balance in Sue's checkbook was −12.55. The balance in John's checkbook was −10.45. Since −12.55 < −10.45, Sue owes more than John. The interpretation could also be "John owes less than Sue."

- Students understand absolute value as the distance from 0 and recognize the symbols | | as representing absolute value. For example, | − 7| can be interpreted as the distance −7 is from 0, which would be 7. Likewise |7| can be interpreted as the distance 7 is from 0, which would also be 7. In real-world contexts, the absolute value can be used to describe size or magnitude. For example, for an ocean depth of $-900 \; feet$, write |−900| = 900 to describe the distance below sea level.

- When working with positive numbers, the absolute value (distance from 0) of the number and the value of the number is the same; therefore, ordering is not problematic. However, negative numbers have a distinction that students need to understand. As the negative number increases (moves to the left on a number line), the value of the number decreases. For example, −24 is less than −14 because −24 is located to the left of −14 on the number line. However, absolute value is the distance from 0. In terms of absolute value (or distance) the absolute value of −24 is greater than −14. For negative numbers, as the absolute value increases, the value of the number decreases.

Standard
6.NS.8 – Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate. *Example:* • If the points on the coordinate plane below are the three vertices of a rectangle, what are the coordinates of the fourth vertex? How do you know? What are the length and width of the rectangle? To determine the distance along the x-axis between the point $(-4, 2)$ and $(2, 2)$ a student must recognize that -4 is $\lvert-4\rvert$ or 4 units to the left of 0 and 2 is $\lvert 2\rvert$ or 2 units to the right of 0, so the two points are a total of 6 units apart along the x-axis. Students should represent this on the coordinate grid and numerically with an absolute value expression, $\lvert-4\rvert + \lvert 2\rvert$.

Coordinate plane with points labeled $(-4, 2)$, $(2, 2)$, and $(-4, -3)$.

Expressions and Equations (EE)

Apply and extend previous understandings of arithmetic to algebraic expressions.

Standard
6.EE.1 – Write and evaluate numerical expressions involving whole-number exponents.   *Examples:*   • Write the following as a numerical expressions using exponential notation. 　▪ The area of a square with a side length of 8 m – solution: $8^2\ m^2 = 64m^2$ 　▪ The volume of a cube with a side length of 5 ft – solution: $5^3 ft^3 = 125 ft^3$ 　▪ Yu-Lee has a pair of mice. The mice each have 2 babies. The babies grow up and have two babies of their own – solution: $2^3\ mice = 8\ mice$ • Evaluate: 　▪ 4^3: solution is 64 　▪ $5 + 2^4 \times 6$: solution is 101 　▪ $7^2 - 24 \div 3 + 26$: solution is 67
6.EE.2 – Write, read, and evaluate expressions in which letters stand for numbers.   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5 – y.*   b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*   c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas V=s³ and A=6s² to find the volume and surface area of a cube with sides of length s=1/2.*   *Explanations:*   • It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number. 　▪ $r + 21$ as "some number plus 21" as well as "r plus 21" 　▪ $n \times 6$ as "some number times 6" as well as "n times 6" 　▪ $\frac{s}{6}$ and $s \div 6$ as "some number divided by 6" as well as "s divided by 6"   • Students should identify the parts of an algebraic expression including variables, coefficients, constants, and the names of operations (sum, difference, product, and quotient). Development of this common language helps students to understand the structure of expressions and explain their process for simplifying expressions.

Standard
6.EE.2 Explanations (continued)

Terms are the parts of a sum. When the term is an explicit number, it is called a constant. When the term is a product of a number and a variable, the number is called the coefficient of the variable.

Variables are letters that represent numbers. There are various possibilities for the numbers they can represent; students can substitute these possible numbers for the letters in the expression for various different purposes.

- Consider the following expression: $x^2 + 5y + 3x + 6$
  - The variables are x and y.
  - There are 3 variable terms—x^2, $5y$, $3x$. They have coefficients of 1, 5, and 3, respectively. The coefficient of x^2 is 1, since $x^2 = 1x^2$. The term "$5y$" represents 5 y's or $5 \times y$.
  - There is one constant term—6.
  - The expression shows a sum of all four terms.

Examples:

- 7 more than 3 times a number – solution: $3x + 7$
- 3 times the sum of a number and 5 – solution: $3(x + 5)$
- 7 less than the product of 2 and a number – solution: $2x - 7$
- Twice the difference between a number and 5 – solution: $2(z - 5)$
- Evaluate $5(n + 3) - 7n$, when $n = \dfrac{1}{2}$
- The expression $c + 0.07c$ can be used to find the total cost of an item with 7% sales tax, where c is the pretax cost of the item. Use the expression to find the total cost of an item that cost $25.
- The perimeter of a parallelogram is found using the formula $p = 2l + 2w$. What is the perimeter of a rectangular picture frame with dimensions of 8.5 inches by 11 inches?

Standard

6.EE.3 – Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

Explanation:

- Students use their understanding of multiplication to interpret $3(2 + x)$. For example, 3 groups of $(2 + x)$. They use a model to represent x, and make an array to show the meaning of $3(2 + x)$. They can explain why it makes sense that $3(2 + x)$ is equal to $6 + 3x$.

  An array with 3 columns and $x + 2$ in each column:
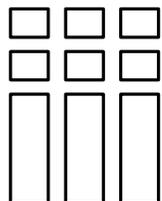
- Students interpret y as referring to one y. Thus, they can reason that one y plus one y plus one y **must be** $3y$. They also use the distributive property, the multiplicative identity property of 1, and the commutative property for multiplication to prove that $y + y + y = 3y$:

  $y + y + y = y \times 1 + y \times 1 + y \times 1 = y \times (1 + 1 + 1) = y \times 3 = 3y$

6.EE.4 – Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Explanation:

- Students demonstrate an understanding of like terms as quantities being added or subtracted with the same variables and exponents. For example, $3x + 4x$ are like terms and can be combined as $7x$; however, $3x + 4 \times 2$ are not. This concept can be illustrated by substituting in a value for x. For example, $9x - 3x = 6x$, not 6. Choosing a value for x, such as 2, can prove non-equivalence.

Example:

- The students in Mr. Nolan's class are writing expressions for the perimeter of a rectangle of side length l and width w. After they share their answers, the following expressions are on the board:
  - Sam: $2(l + w)$
  - Joanna: $l + w + l + w$
  - Kiyo: $2l + w$
  - Erica: $2w + 2l$
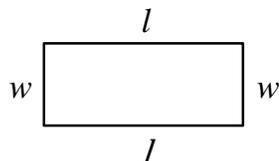
  Which of the expressions are correct, and how might the students have been thinking about finding the perimeter of the rectangle?

Reason about and solve one-variable equations and inequalities.

Standard
6.EE.5 – Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Explanation:

- Students identify values from a specified set that will make an equation true. For example, given the expression $x + 2\frac{1}{2}$, which of the following value(s) for x would make $x + 2\frac{1}{2} = 6$.

$\{0,\ 3\frac{1}{2},\ 4\}$

By using substitution, students identify $3\frac{1}{2}$ as the value that will make both sides of the equation equal.

The solving of inequalities is limited to choosing values from a specified set that would make the inequality true. For example, find the value(s) of x that will make $x + 3.5 \geq 9$.

$\{5,\ 5.5,\ 6, \frac{15}{2}, 10.2, 15\}$

Using substitution, students identify $5.5,\ 6,\ \frac{15}{2}, 10.2,$ and 15 as the values that make the inequality true. Note: If the inequality had been $x + 3.5 > 9$, then 5.5 would not work since 9 is not greater than 9.

Standard

6.EE.6 – Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

Explanation: Connecting writing expressions with story problems and/or drawing pictures will give students a context for this work. It is important for students to read algebraic expressions in a manner that reinforces that the variable represents a number.

Examples:

- Maria has three more than twice as many crayons as Elizabeth. Write an algebraic expression to represent the number of crayons that Maria has.
  - Solution: $2c + 3$, where c represents the number of crayons that Elizabeth has.
- An amusement park charges $28.00 to enter and $0.35 per ticket. Write an algebraic expression to represent the total amount spent.
  - Solution: $28.00 + 0.35t$, where t represents the number of tickets purchased.
- Andrew has a summer job doing yard work. He is paid $15 per hour and a $20 bonus when he completes the yard. He was paid $85 for completing one yard. Write an equation to represent the amount of money he earned.
  - Solution: $15h + 20 = 85$, where h is the number of hours worked.
- Describe a problem situation that can be solved using the equation $2c + 3 = 15$, where c represents the cost of an item.
- Bill earned $5.00 mowing the lawn on Saturday. He earned more money on Sunday. Write an expression that shows the amount of money Bill has earned.
  - Solution: $5.00 + n$

| Standard |
| --- |
| **6.EE.7 –** Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which are all positive rational numbers.

Explanation:

• Students create and solve equations that are based on real world situations. It may be beneficial for students to draw pictures that illustrate the equation in problem situations. Solving equations using reasoning and prior knowledge should be required of students to allow them to develop effective strategies.

Example:

• Meagan spent $56.58 on three pairs of jeans. If each pair of jeans costs the same amount, write an algebraic equation that represents this situation and solve to determine how much one pair of jeans cost.

<table><tr><td colspan="3" align="center">$56.58</td></tr><tr><td align="center">J</td><td align="center">J</td><td align="center">J</td></tr></table>
Sample Solution: Students might say, "I created the bar model to show the cost of the three pairs of jeans. Each bar labeled J is the same size because each pair of jeans costs the same amount of money. The bar model represents the equation $3J = \$56.58$. To solve the problem, I need to divide the total cost of $56.58 between the three pairs of jeans. I know that it will be more than $10 each because 10×3 is only $30, but it will be less than $20 each because 20×3 is $60. If I start with $15 each, I am up to $45. I have $11.58 left. I then give each pair of jeans $3. That is $9 more dollars. I only have $2.58 left. I continue until all the money is divided. I ended up giving each pair of jeans another $0.86. Each pair of jeans costs $18.86 ($15 + $3 + $0.86). I double check that the jeans cost $18.86 each because $18.86 \times 3 = \$56.58$."

• Julio gets paid $20 for babysitting. He spends $1.99 on a package of trading crds and $6.50 on lunch. Write and solve an equation to show how much money Julio has left.

▪ Solution: $20 = \$1.99 + \$6.50 + x$, therefore $x = \$11.51$

<table><tr><td colspan="3" align="center">$20</td></tr><tr><td align="center">$1.99</td><td align="center">$6.50</td><td align="center">money left over (x)</td></tr></table> |

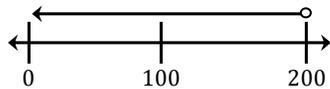| Standard |
| --- |
| **6.EE.8 –** Write an inequality of the form *x* > *c* or *x*< *c* to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form *x* > *c* or *x* < *c* have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Explanation: A number line diagram is drawn with an open circle when an inequality contains a < or > symbol to show solutions that are less than or greater than the number but not equal to the number. The circle is shaded, as in the first example below, when the number is to be included. Students recognize that possible values can include fractions and decimals, which are represented on the number line by shading. Shading is extended through the arrow on a number line to show that an inequality has an infinite number of solutions.

Examples:

• Graph $x \leq 4$
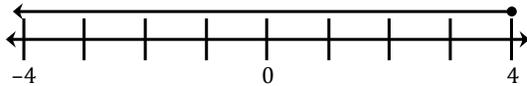
• Jonas spent more than \$50 at an amusement park. Write an inequality to represent the amount of money Jonas spent. What are some possible amounts of money Jonas could have spent? Represent the situation on a number line.

• Less than \$200.00 was spent by the Flores family on groceries last month. Write an inequality to represent this amount and graph this inequality on a number line.
  ▪ Solution: $200 > x$

 |

Represent and analyze quantitative relationships between dependent and independent variables.

Standard

6.EE.9 – Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Explanation: Students can use many forms to represent relationships between quantities. Multiple representations include describing the relationship using language, a table, an equation, or a graph. Translating between multiple representations helps students understand that each form represents the same relationship and provides a different perspective on the function.

Examples:

- What is the relationship between the two variables? Write an expression that illustrates the relationship.

x	1	2	3	4
y	2.5	5	7.5	10

- Use the graph below to describe the change in y as x increases by 1.
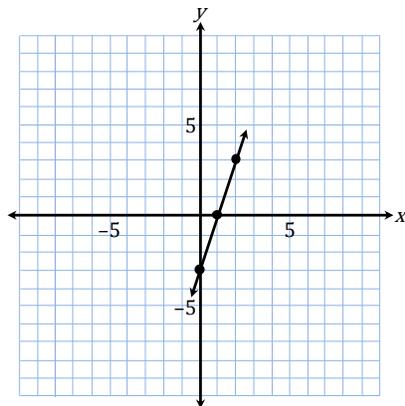
6.EE.9 Examples (continued)

- Susan started with $1 in her savings. She plans to add $4 per week to her savings. Use an equation, table, and graph to demonstrate the relationship between the number of weeks that pass and the amount in her savings account.

  - Language: Susan has $1 in her savings account (s). She is going to save $4 each week ($w$).

  - Equation: $s = 4w + 1$

  - Table:

s	w
0	1
1	5
2	9

  - Graph:
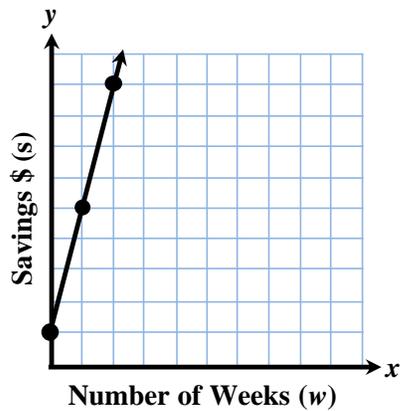
Geometry (G)

***Solve real-world and mathematical problems involving area, surface area, and volume.***

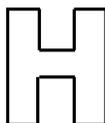| Standard |
|---|
| **6.G.1 –** Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Resources: Special quadrilaterals include rectangles, squares, parallelograms, trapezoids, rhombi, and kites. Students can use tools such as the Isometric Drawing Tool on [NCTM's Illuminations](#) site to shift, rotate, color, decompose, and view figures in 2D or 3D.

Examples:
• Find the area of a triangle with a base length of three units and a height of four units.
• Find the area of the trapezoid shown below using the formulas for rectangles and triangles.

![trapezoid with top 12, height 3, bottom 7]
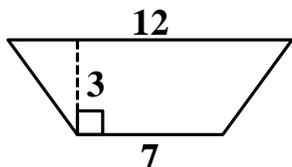
• A rectangle measures 3 inches by 4 inches. If the lengths of each side double, what is the effect on the area?
• The area of the rectangular school garden is 24 square units. The length of the garden is 8 units. What is the length of the fence needed to enclose the entire garden?
• The 6th grade class at Hernandez School is building a giant wooden H for their school. The H will be 10 feet tall and 10 feet wide and the thickness of the block letter will be 2.5 feet.
    ▪ How large will the H be if measured in square feet?
    ▪ The truck that will be used to bring the wood from the lumberyard to the school can only hold a piece of wood that is 60 inches by 60 inches. What pieces of wood (how many pieces and what dimensions) are needed to complete the project?

![block letter H] |

Standard
6.G.2 – Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas *V = lwh* and *V = bh* to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

Resources: Students need multiple opportunities to measure volume by filling rectangular prisms with blocks and looking at the relationship between the total volume and the area of the base. Through these experiences, students derive the volume formula (volume equals the area of the base times the height). Students can explore the connection between filling a box with unit cubes and the volume formula using interactive applets such as the Cubes Tool on NCTM's Illuminations.

In addition to filling boxes, students can draw diagrams to represent fractional side lengths, connecting with multiplication of fractions. This process is similar to composing and decomposing two-dimensional shapes.

Examples:

- The model shows a cubic foot filled with cubic inches. The cubic inches can also be labeled as a fractional cubic unit with dimensions of $\frac{1}{12} ft^3$.
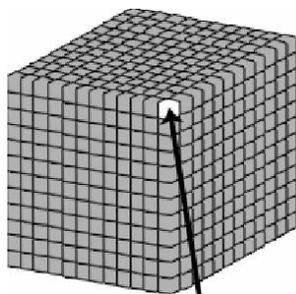
$$\frac{1}{12} \text{ of } \frac{1}{12} \text{ of } \frac{1}{12}$$

- The models show a rectangular prism with dimensions $\frac{3}{2}$ inches, $\frac{5}{2}$ inches, and $\frac{5}{2}$ inches. Each of the cubic units in the model is $\frac{1}{2} in^3$. Students work with the model to illustrate $\frac{3}{2} \times \frac{5}{2} \times \frac{5}{2} = (3 \times 5 \times 5) \times \frac{1}{8}$. Students reason that a small cube has volume $\frac{1}{8}$ because 8 of them fit in a unit cube.
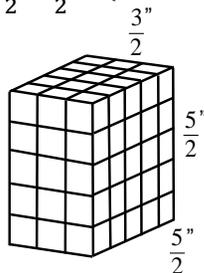
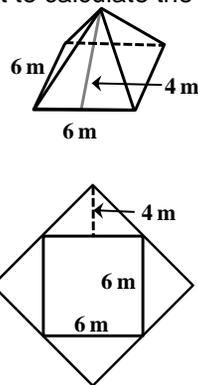| Standard |
| --- |
| **6.G.3 –** Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.

Example:

• On a map, the library is located at $(-2, 2)$, the city hall building is located at $(0, 2)$, and the high school is located at $(0, 0)$. Represent the locations as points on a coordinate grid with a unit of 1 mile.

 ▪ What is the distance from the library to the city hall building? The distance from the city hall building to the high school? How do you know?

 ▪ What shape is formed by connecting the three locations? The city council is planning to place a city park in this area. How large is the area of the planned park? |
| **6.G.4 –** Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Resources:

• Students construct models and nets of three-dimensional figures, describing them by the number of edges, vertices, and faces. Solids include rectangular and triangular prisms. Students are expected to use the net to calculate the surface area.

• Students can create nets of three-dimensional figures with specified dimensions using the Dynamic Paper Tool on NCTM's Illuminations.

• Students also describe the types of faces needed to create a three-dimensional figure. Students make and test conjectures by determining what is needed to create a specific three-dimensional figure.

Examples:

• Describe the shapes of the faces needed to construct a rectangular pyramid. Cut out the shapes and create a model. Did your faces work? Why or why not?

• Create the net for a given prism or pyramid and then use the net to calculate the surface area.

6 m 4 m 6 m

4 m 6 m 6 m |

Delaware Department of Education

Statistics and Probability (SP)

Develop understanding of statistical variability.

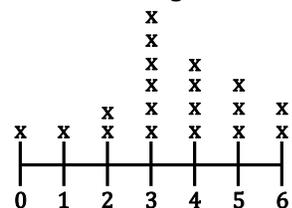| Standard |
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| **6.SP.1 –** Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*

Explanation:

• Statistics are numerical data relating to an aggregate of individuals; statistics is also the name for the science of collecting, analyzing, and interpreting such data. A statistical question anticipates an answer that varies from one individual to the next and is written to account for the variability in the data. Data are the numbers produced in response to a statistical question. Data are frequently collected from surveys or other sources (i.e., documents).

Questions can result in a narrow or wide range of numerical values. For example, asking classmates, "How old are the students in my class in years?" will result in less variability than asking "How old are the students in my class in months?"

Students might want to know about the fitness of the students at their school. Specifically, they want to know about the exercise habits of the students. So rather than asking, "Do you exercise?" they should ask about the amount of exercise the students at their school get per week. A statistical question for this study could be, "How many hours per week on average do students at your middle school exercise?"

To collect this information, students might design a survey question that anticipates variability by providing a variety of possible anticipated responses that have numerical answers, such as 3 hours per week, 4 hours per week, and so on. Be sure that students ask questions that have specific numerical answers. |

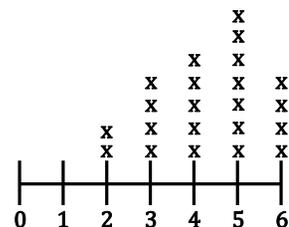| Standard |
| --- |
| **6.SP.2 –** Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.

Example:

• The two dot plots show the 6-trait writing scores for a group of students on two different traits—organization and ideas. The center, spread and overall shape can be used to compare the data sets. Students consider the context in which the data were collected and identify clusters, peaks, gaps, and symmetry. Showing the two graphs vertically rather than side by side helps students make comparisons. For example, students would be able to see from the display of the two graphs that the ideas scores are generally higher than the organization scores. One observation students might make is that the scores for organization are clustered around a score of 3 whereas the scores for ideas are clustered around a score of 5. |

6-Trait Writing Rubric
Scores for Organization

```
              x
              x
              x   x
              x   x   x
          x   x   x   x   x
  x   x   x   x   x   x   x
  |---|---|---|---|---|---|
  0   1   2   3   4   5   6
```

6-Trait Writing Rubric
Scores for Ideas

```
                  x
                  x
              x   x
          x   x   x   x
          x   x   x   x
      x   x   x   x   x
      x   x   x   x   x
  |---|---|---|---|---|---|
  0   1   2   3   4   5   6
```
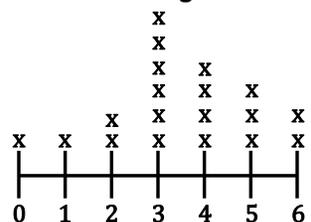
Standard

6.SP.3 – Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Explanation: When using measures of center (mean, median, and mode) and range, students are describing a data set in a single number. The range provides a single number that describes how the values vary across the data set. The range can also be expressed by stating the minimum and maximum values.

Example:

- Consider the data shown in the dot plot of the 6-trait scores for organization for a group of students.
  - How many students are represented in the data set?
  - What are the mean, median, and mode of the data set? What do these values mean? How do they compare?
  - What is the range of the data? What does this value mean?

6-Trait Writing Rubric
Scores for Organization

```
                    x
                    x
                    x   x
                    x   x   x
                x   x   x   x   x
    x   x   x   x   x   x   x
    |   |   |   |   |   |   |
    0   1   2   3   4   5   6
```

Delaware Department of Education

Summarize and describe distributions.

Standard
6.SP.4 – Display numerical data in plots on a number line, including dot plots, histograms, and box plots.

Resources: In order to display numerical data in dot plots, histograms, or box plots, students need to make decisions and perform calculations. Students are expected to display data graphically in a format appropriate for that data set as well as reading data from graphs generated by other students or contained in reference materials. Students can use applets to create data displays. Examples of applets include the Box Plot Tool and Histogram Tool on NCTM's Illuminations.

Explanations:

- Dot plots are simple plots on a number line where each dot represents a piece of data in the data set. Dot plots are suitable for small to moderate size data sets and are useful for highlighting the distribution of the data including clusters, gaps, and outliers.
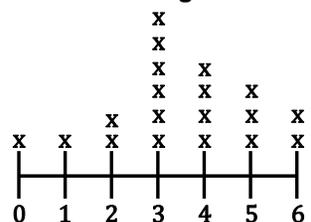
  In most real data sets, there is a large amount of data and many numbers will be unique. A graph, such as a dot plot, that shows how many ones, how many twos, etc., would not be meaningful; however, a histogram can be used. Students organize the data into convenient ranges and use these intervals to generate a frequency table and histogram. Note that changing the size of the range changes the appearance of the graph and the conclusions you may draw from it.

  Box plots are another useful way to display data and are plotted horizontally or vertically on a number line. Box plots are generated from the five number summary of a data set consisting of the minimum, maximum, median, and two quartile values. Students can readily compare two sets of data if they are displayed with side-by-side box plots on the same scale. Box plots display the degree of spread of the data and the skewness of the data.

Examples:

- Nineteen students completed a writing sample that was scored using the 6-traits rubric. The scores for the trait of organization were 0, 1, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6. Create a data display. What are some observations that can be made from the data display?

**6-Trait Writing Rubric**
Scores for Organization

```
                x
                x
            x   x
            x   x   x
        x   x   x   x   x
    x   x   x   x   x   x   x
    |   |   |   |   |   |   |
    0   1   2   3   4   5   6
```

Standard

6.SP.4 Examples (continued)

- Grade 6 students were collecting data for a math class project. They decided they would survey the other two grade 6 classes to determine how many DVDs each student owns. A total of 38 students were surveyed. The data are shown in the table below in no specific order. Create a data display. What are some observations that can be made from the data display?

11	21	5	12	10	31	19	13	23	33
10	11	25	14	34	15	14	29	8	5
22	26	23	12	27	4	25	15	7	
2	19	12	39	17	16	15	28	16	

A histogram using 5 ranges (0-9, 10-19, 20-29, and 30-39) to organize the data is displayed below.
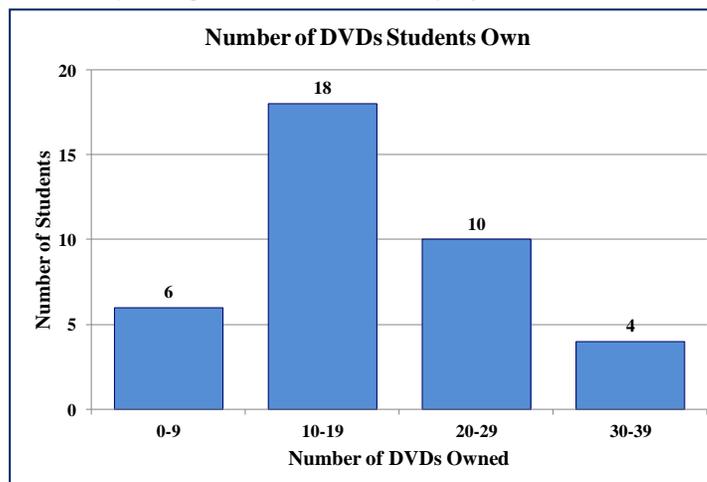
Standard

*6.SP.4 Examples (continued)*

- Ms. Wheeler asked each student in her class to write their age in months on a sticky note. The 28 students in the class brought their sticky note to the front of the room and posted them in order on the white board. The data set is listed below in order from least to greatest. Create a data display. What are some observations that can be made from the data display?

130	130	131	131	132	132	132	133	134	136
137	137	138	139	139	139	140	141	142	142
142	143	143	144	145	147	149	150		

Five-number summary:

- Minimum – 130 months
- Quartile 1 ($Q1$) – $(132 + 133) \div 2 = 132.5$ months
- Median ($Q2$) – 139 months
- Quartile 3 ($Q3$) – $(142 + 143) \div 2 = 142.5$ months
- Maximum – 150 months

Ages in Months of a Class of 6th Grade Students
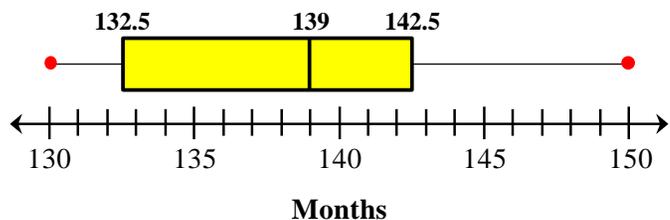
Months

This box plot shows that:

- $\frac{1}{4}$ of the students in the class are from 130 to 132.5 months old
- $\frac{1}{4}$ of the students in the class are from 142.5 months to 150 months old
- $\frac{1}{2}$ of the class are from 132.5 to 142.5 months old
- The median class age is 139 months

Standard

**6.SP.5 –** Summarize numerical data sets in relation to their context, such as by:

a. Reporting the number of observations.

b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.

c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation) as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.

d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Explanations: The measure of center that a student chooses to describe a data set will depend upon the shape of the data distribution and context of data collection. The mode is the value in the data set that occurs most frequently. The mode is the least frequently used as a measure of center because data sets may not have a mode, may have more than one mode, or the mode may not be descriptive of the data set. The mean is a very common measure of center computed by adding all the numbers in the set and dividing by the number of values. The mean can be affected greatly by a few data points that are very low or very high. In this case, the median, or middle value of the data set, might be more descriptive. In data sets that are symmetrically distributed, the mean and median will be very close to the same. In data sets that are skewed, the mean and median will be different, with the median frequently providing a better overall description of the data set.

- Understanding the Mean – The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students develop understanding of what the mean represents by redistributing data sets to be level or fair. The leveling process can be connected to and used to develop understanding of the computation of the mean.

  Example:

  - Students can generate a data set by measuring the number of jumping jacks they can perform in 5 seconds, the length of their feet to the nearest inch, or the number of letters in their names. It is best if the data generated for this activity are 5 to 10 data points which are whole numbers between 1 and 10 that are easy to model with counters or stacking cubes. Students generate a data set by drawing 8 student names at random from the popsicle stick cup. The number of letters in each of the names is used to create the data set. If the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen, there would be 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters, and 1 name with 7 letters. This data set could be represented with stacking cubes.
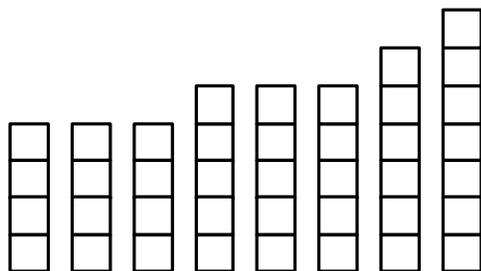
Standard

6.SP.5 Examples (continued)

Students can model the mean by "leveling" the stacks or distributing the blocks so the stacks are "fair." Students are seeking to answer the question "If all of the students had the same number of letters in their name, how many letters would each person have?"

One block from the stack of 6 and 2 blocks from the stack of 7 can be moved down to the stacks of 4 and then all the stacks have 5 blocks. If all students had the same number of letters in their name, they would have five letters. The mean number of letters in a name in this data set is 5.
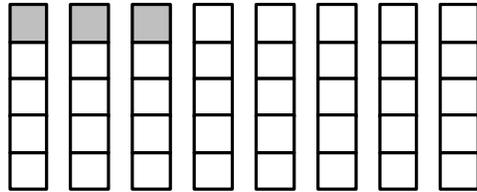

If it was not possible to make the stacks exactly even, students could begin to consider what part of the extra blocks each stack would have.

- Understanding Mean Absolute Deviation – The use of mean absolute deviation in 6th grade is mainly exploratory. The intent is to build a deeper understanding of variability. Students would understand the mean distance between the pieces of data and the mean of the data set expresses the spread of the data set. Students can see that the larger the mean distance, the greater the variability. Comparisons can be made between different data sets.

Example:

- In the previous data set, the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. There were 3 names with 4 letters each, 3 names with 5 letters each, 1 name with 6 letters, and 1 name with 7 letters. This data can be represented on a dot plot. The mean of the data set is 5.

Number of Letters in Name
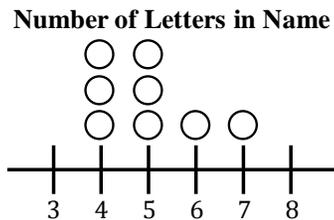
Standard

6.SP.5 Examples (continued)

To find the mean absolute deviation, students examine each of the data points and its difference from the mean. This analysis can be represented on the dot plot itself or in a table. Each of the names with 4 letters has 1 fewer letter than the mean, each of the names with 5 letters has 0 difference in letters as compared to the mean, each of the names with 6 letters has 1 more letter than the mean, and each of the names with 7 letters has 2 more letters than the mean. The absolute deviations are the absolute value of each difference.
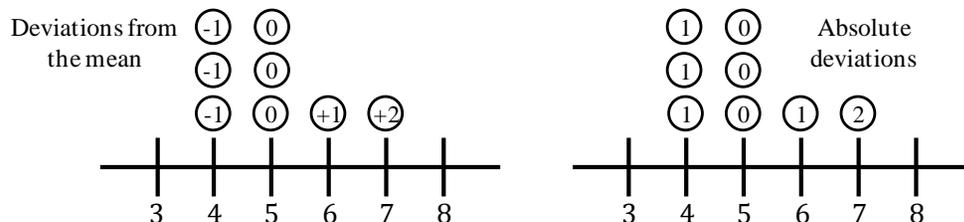
Name	Number of Letters in Name	Deviation from Mean	Absolute Deviation from Mean
John	4	-1	1
Luis	4	-1	1
Mike	4	-1	1
Carol	5	0	0
Maria	5	0	0
Karen	5	0	0
Sierra	6	+1	1
Monique	7	+2	2
Total	40	0	6

The mean of the absolute deviations is found by summing the absolute deviations and dividing by the number of data points. In this case, the mean absolute deviation would be $6 \div 8$ or $\frac{3}{4}$ or 0.75. The mean absolute deviation is a small number, indicating that there is little variability in the data set.

Consider a different data set also containing 8 names. If the names were Sue, Joe, Jim, Amy, Sabrina, Monique, Timothy, and Adelita, summarize the data set and its variability. How does this compare to the first data set?

The mean of this data set is still 5: $\frac{(3+3+3+3+7+7+7+7)}{8} = \frac{40}{8} = 5$

6.SP.5 Examples (continued)

Name	Number of Letters in Name	Deviation from Mean	Absolute Deviation from Mean
Sue	3	-1	1
Joe	3	-1	1
Jim	3	-1	1
Amy	3	0	0
Sabrina	7	0	0
Timothy	7	0	0
Adelita	7	+1	1
Monique	7	+2	2
Total	40	0	6

The mean deviation of this data set is $16 \div 8$ or 2. Although the mean is the same, there is much more variability in this data set.

- Understanding Medians and Quartiles – Students can also summarize and describe the center and variability in data sets using the median and a five number summary consisting of the minimum, quartiles, and maximum as seen in the box plot example in 6.SP.4. The median is the middle number of the data set with half the numbers below the median and half the numbers above the median. The quartiles partition the data set into four parts by dividing each of the halves of the data set into half again. Quartile 1 ($Q1$ or the lower quartile) is the middle value of the lower half of the data set and quartile 3 ($Q3$ or the upper quartile) is the middle value of the upper half of the data set. The median can also be referred to as quartile 2 ($Q2$). The range of the data is the difference between the minimum and maximum values. The interquartile range of the data is the difference between the lower and upper quartiles ($Q3 - Q1$). The interquartile range is a measure of the dispersion or spread of the data set—a small value indicates values that are clustered near the median whereas a larger value indicates values that are more distributed.

Example:

- Consider the first data set again. Recall that the names drawn were Carol, Mike, Maria, Luis, Monique, Sierra, John, and Karen. The data set can be represented in a numerical list. To find the median and quartile, the values are placed in order from least to greatest.

5 4 5 4 7 6 4 5 ⟶ 4 4 4 5 5 5 6 7

The middle value in the ordered data set is the median. If there is an even number of values, the median is the mean of the middle two values. In this case, the median would be 5 because 5 is the average of the 4th and 5th values, which are both 5. Students find quartile 1 ($Q1$) by examining the lower half of the data. Again there are 4 values, which is an even number of values. $Q1$ would be the average of the 2 and 3 value in the data set or 4. Students find quartile 3 ($Q3$) by examining the upper half of the data. $Q3$ would be the average of the 6th and 7th value in the data set or 5.5. The mean of the data set is 5, and the median is also 5, showing that the values are probably clustered close to the mean. The interquartile range is 1.5 ($5.5 - 4.0$). The interquartile range is small, showing little variability in the data.

4   4   4   5   5   5   6   7
        ↑   ↑   ↑
    $Q1 = 4$ $Q3 = 5.5$
          $Median = 4$